STUDENTS BECOMING ASSESSORS

HANDOUTS FOR TEACHERS

Contents

Acknowledgement:

In preparing this material, we acknowledge the permissions given by the Bowland Charitable Trust to adapt the professional development resources, Bowland Maths (www.bowlandmaths.org), that they had previously funded the University of Nottingham to produce for the UK. This includes many of the handouts and most of the video extracts. Additional resources were also adapted from *Improving Learning in Mathematics*, a government funded program in the UK and from the *Mathematics Assessment Project* funded by the Bill and Melinda Gates Foundation.

Handout 1: Text messaging

1. How many text messages are sent if four people all send messages to each other?

2. How many text messages are sent with different numbers of people?

3. Approximately how many text messages would travel in cyberspace if everyone in your school took part?

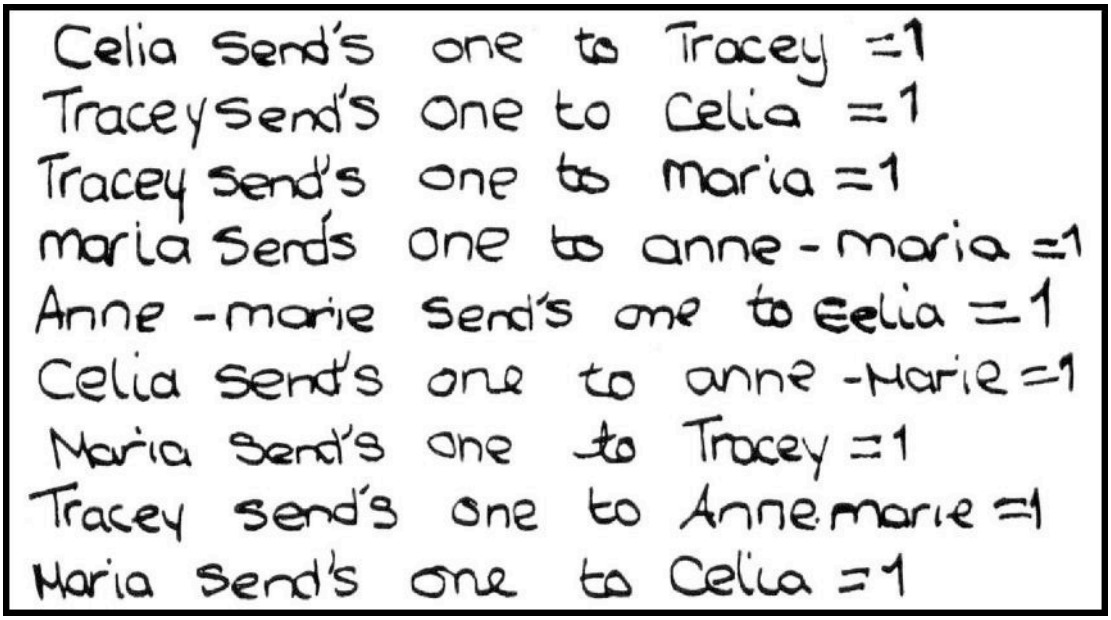
4. Can you think of other situations that would give rise to the same mathematical relationship?

This was adapted from Sending texts – a task from the Nuffield Foundation's Applying Mathematical Processes project – see http://www.nuffieldcurriculumcentre.org/

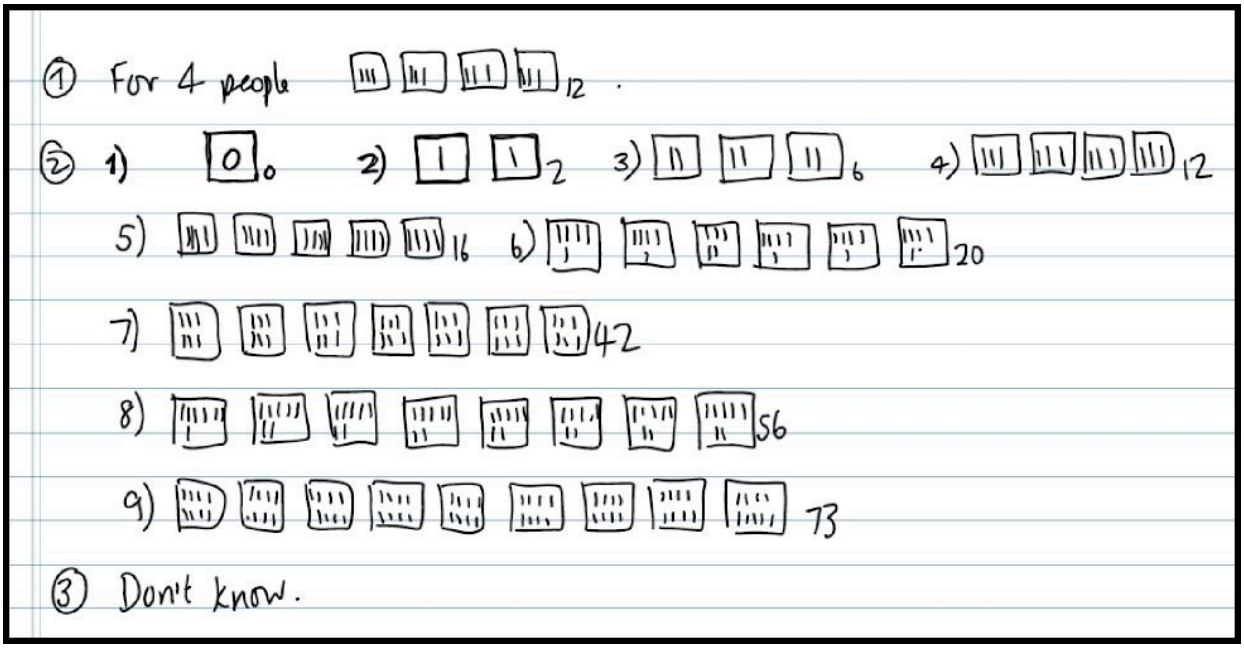
Follow-up task for students

Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

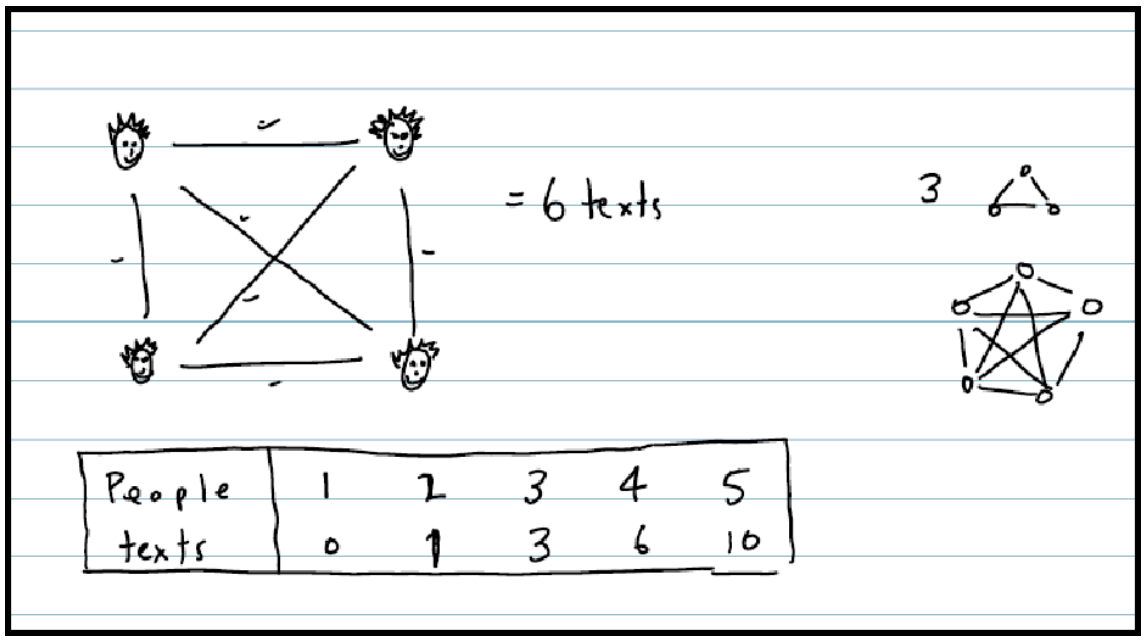
Tom's answer



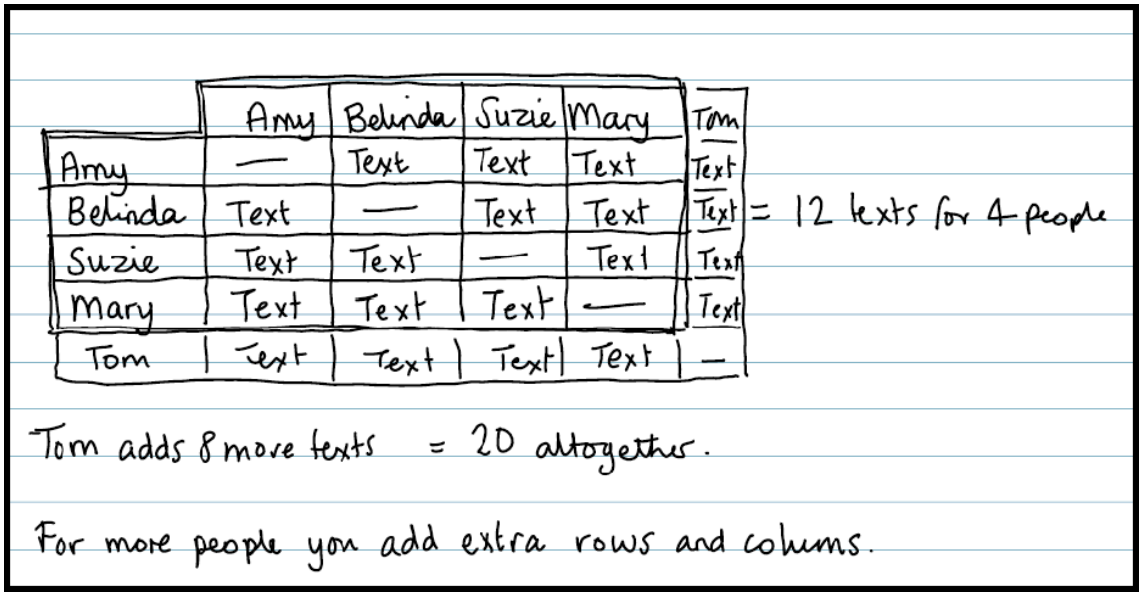
Sam's answer



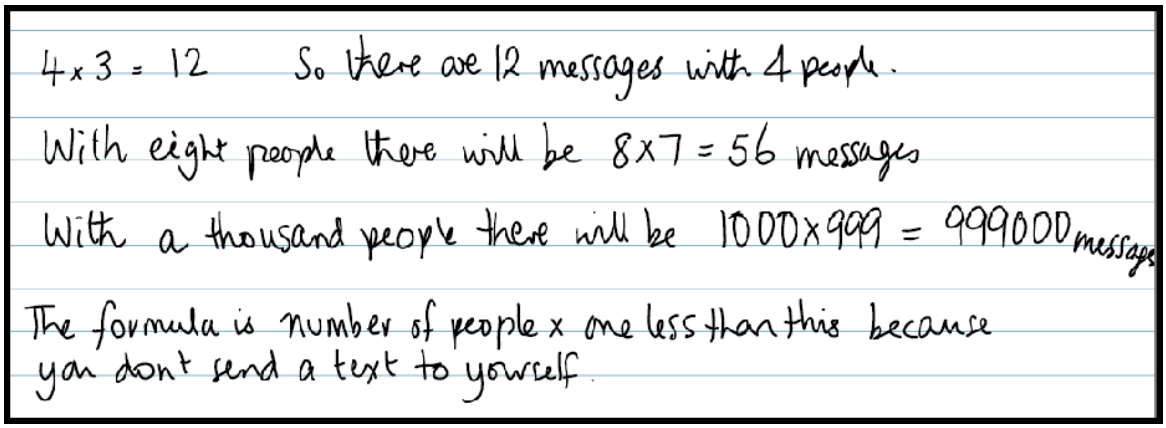
Chris's answer



Lily's answer



Marvin's answer



Reflecting on Sheena's lesson

What aspects of the provided samples of work do students attend to?

What criteria do students use as they assess the sample work?

What are students learning from the sample work?

Concerns about using student work

Comment on Sheena's orchestration of the activity.

What could you do to avoid potential problems when your students assess other students' work?

Choose a topic or lesson that you have recently taught or planned and think about the samples you might use.

Briefly outline the lesson.

What specific examples would be useful?

Why would they be useful?

What criteria would you use for choosing sample work to use with students?

How would you introduce the activity in your class?

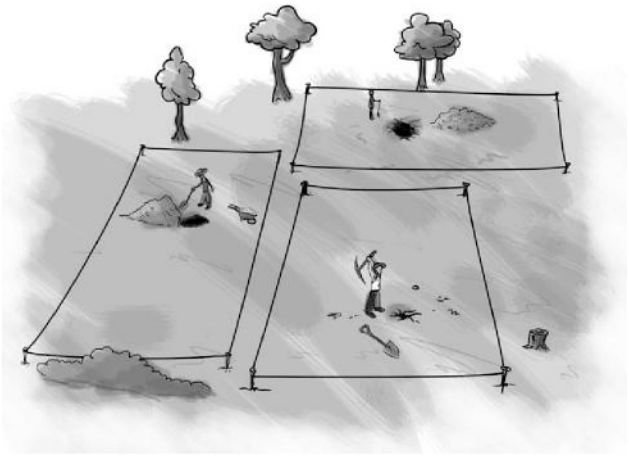
What would you be looking out for as you circulate, in the time that the students are discussing the samples? Why?

Handout 2: Golden Rectangles

In the 19th century, many adventurers travelled to North America to search for gold.

A man named Dan Jackson owned some land where gold had been found.

Instead of digging for the gold himself, he rented plots of land to the adventurers.



Dan gave each adventurer four wooden stakes and a rope measuring exactly 100 metres.

Each adventurer had to use the stakes and the rope to mark off a rectangular plot of land.

1. Assuming each adventurer would like to have the biggest plot, how should he place his stakes?

   Explain your answer.

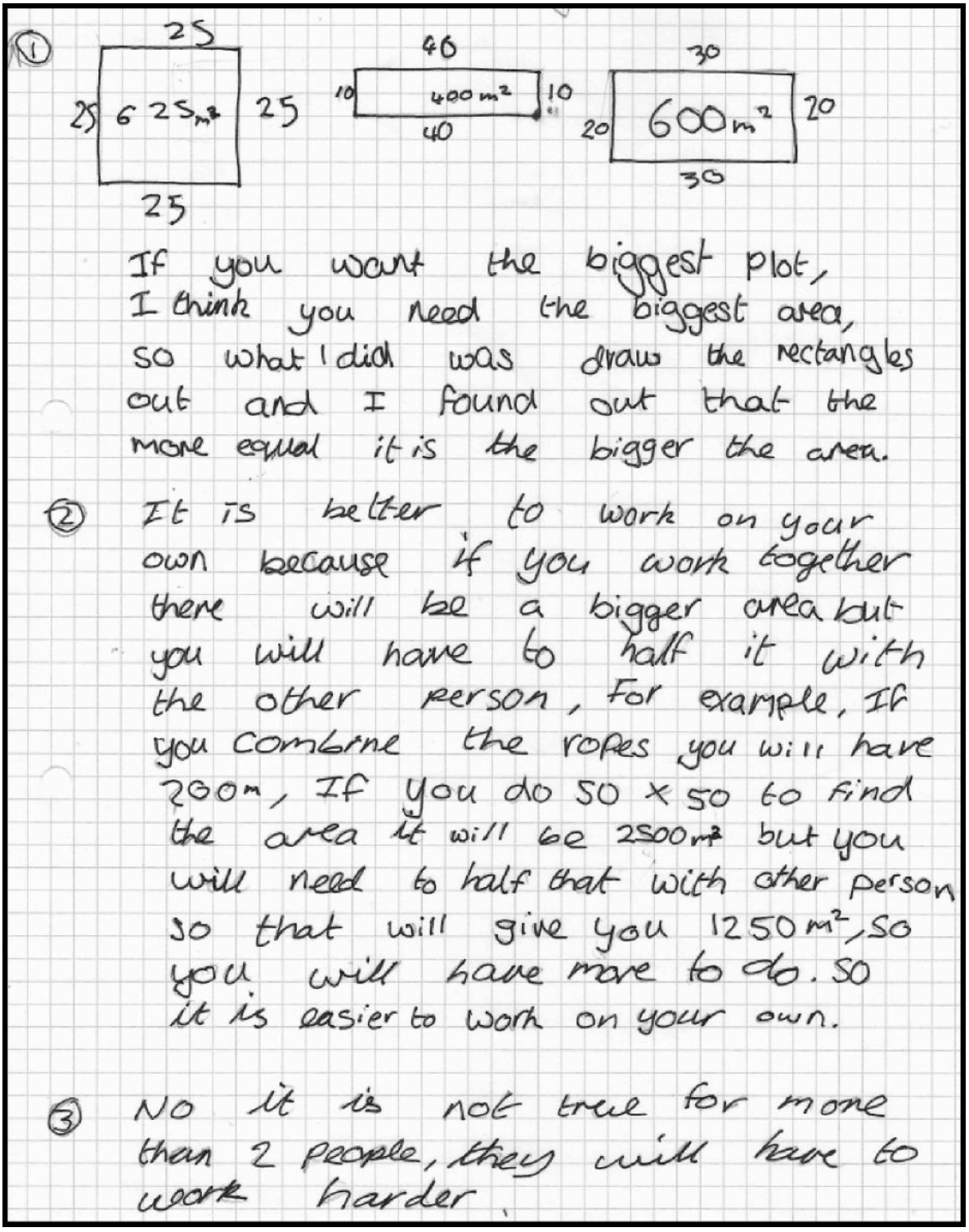
2. Read the following proposition:

   "Tie the ropes together! You can get more land if you work together than if you work separately."

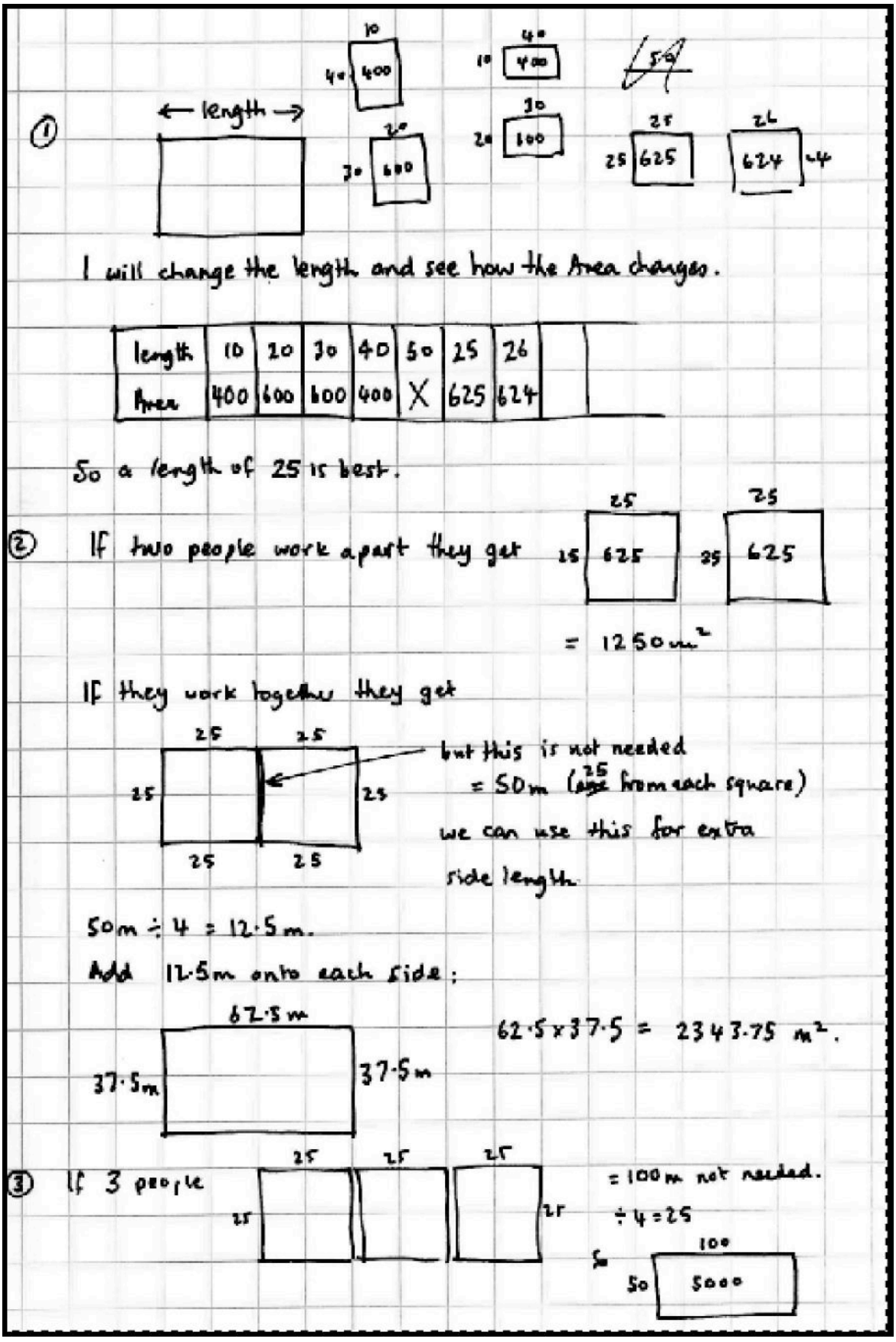
   Investigate whether the proposition is true for two adventurers working, still using four stakes.

3. Is the proposition true for more than two people? Explain your answer.

Alvin's answer



Bernie's answer



Chris's answer

a $25 \times 25 = 625\ m^2$

~~[illegible]~~

$30 \times 20 = 600\ m^2$

$40 \times 10 = 400\ m^2$

He should place the stakes in a rechangulae because then he has the most land. But the rectangle need to be 30×20m.

b With two ropes of 100 m, you can get a bigger amount of land. If you take 55m x 45m, you get more than the dubble amount of land. $55 \times 45 = 2475$.

$2475\ m^2 : 2 = 1237.5\ m^2$

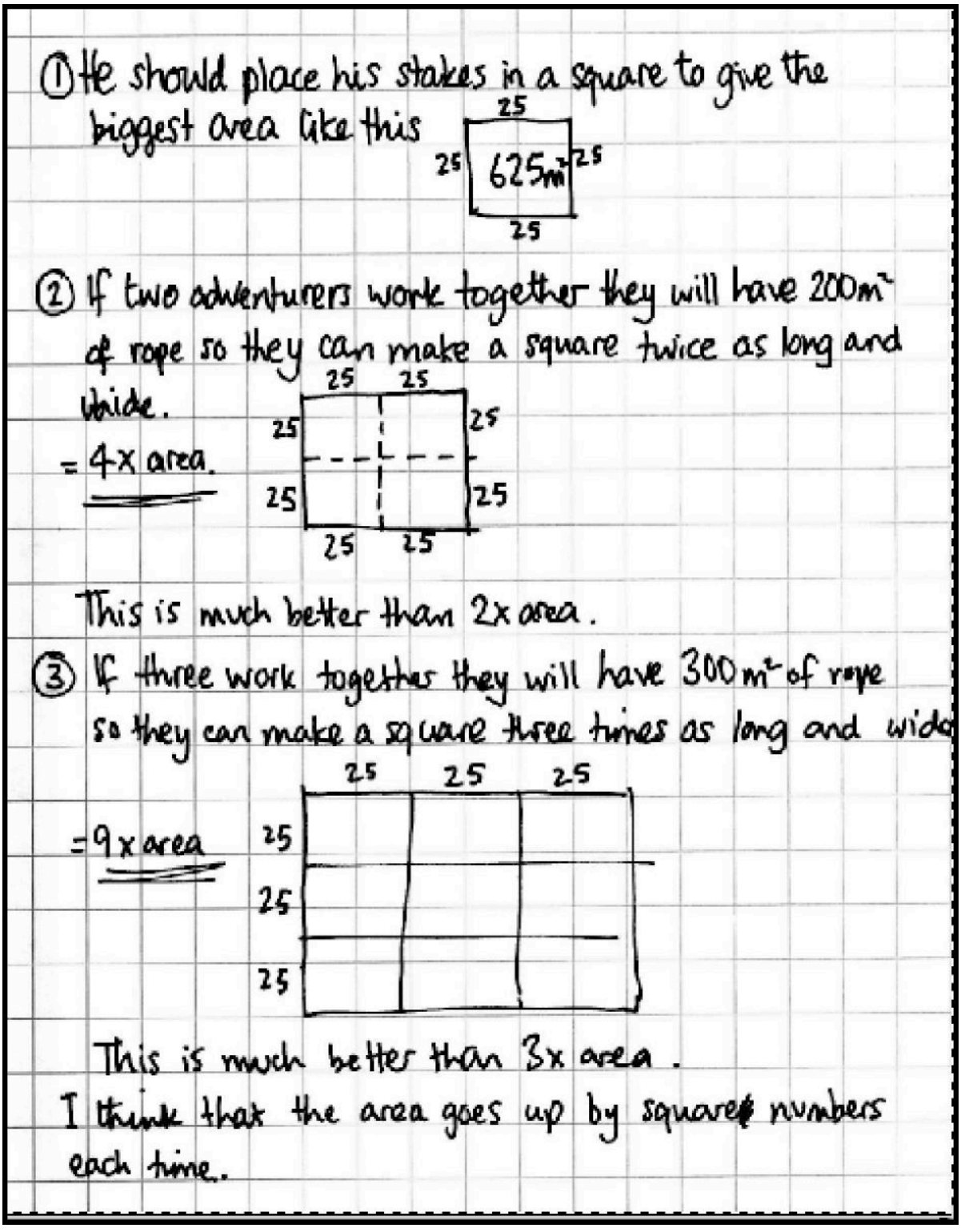
c Yes, because you can make the plot of land bigger in that way everyone has more land.

If the plot of land is 80×70, the land is $5600\ m^2$.

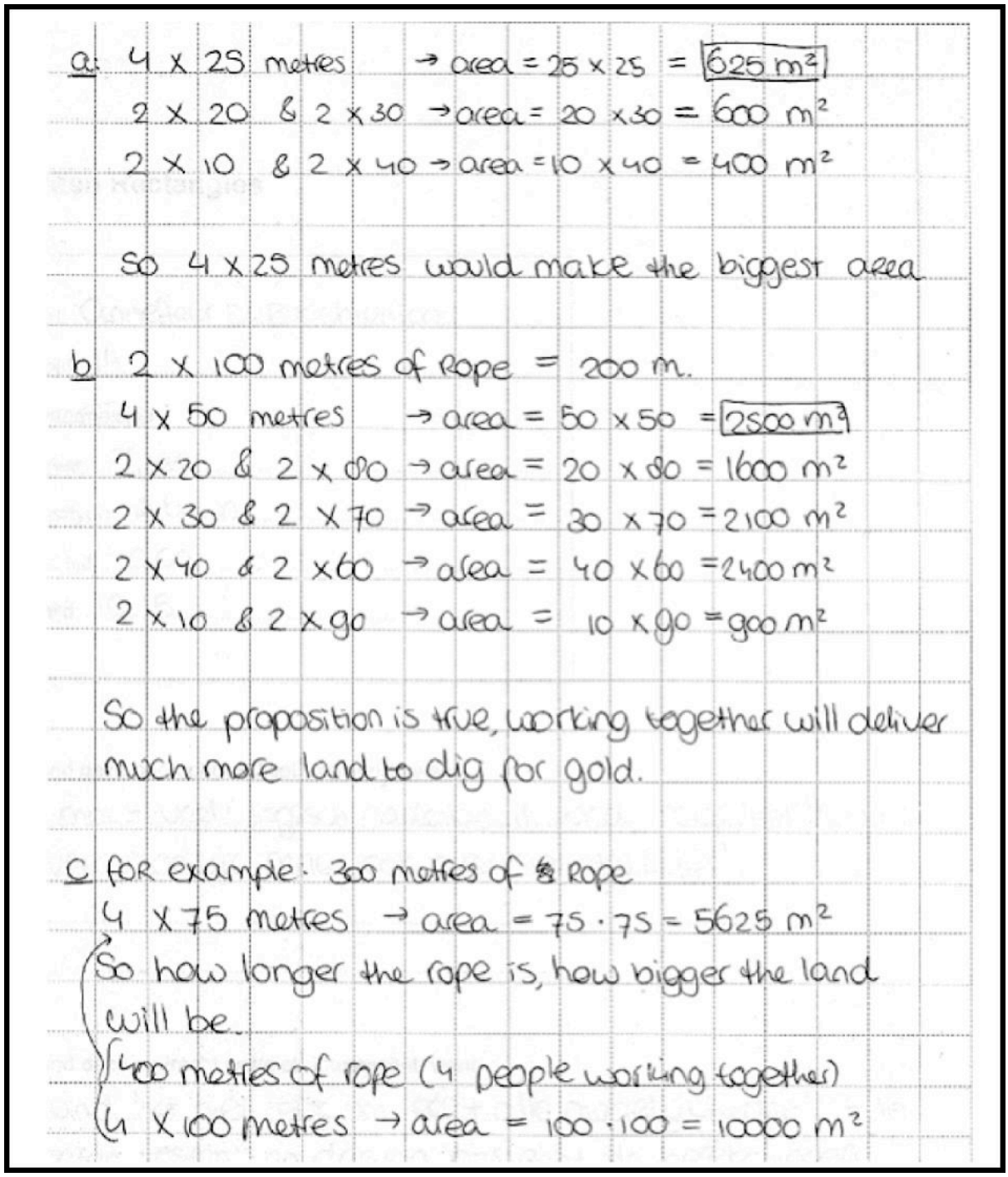
$5600\ m^2 : 3 = 1866.67 \rightarrow 1866.7\ m^2$ per person.

That is more land.

Danny's answer



Elsie's answer



a: 4 x 25 metres → area = 25 x 25 = 625 m²
2 x 20 & 2 x 30 → area = 20 x 30 = 600 m²
2 x 10 & 2 x 40 → area = 10 x 40 = 400 m²

So 4 x 25 metres would make the biggest area

b 2 x 100 metres of rope = 200 m.
4 x 50 metres → area = 50 x 50 = 2500 m²
2 x 20 & 2 x 80 → area = 20 x 80 = 1600 m²
2 x 30 & 2 x 70 → area = 30 x 70 = 2100 m²
2 x 40 & 2 x 60 → area = 40 x 60 = 2400 m²
2 x 10 & 2 x 90 → area = 10 x 90 = 900 m²

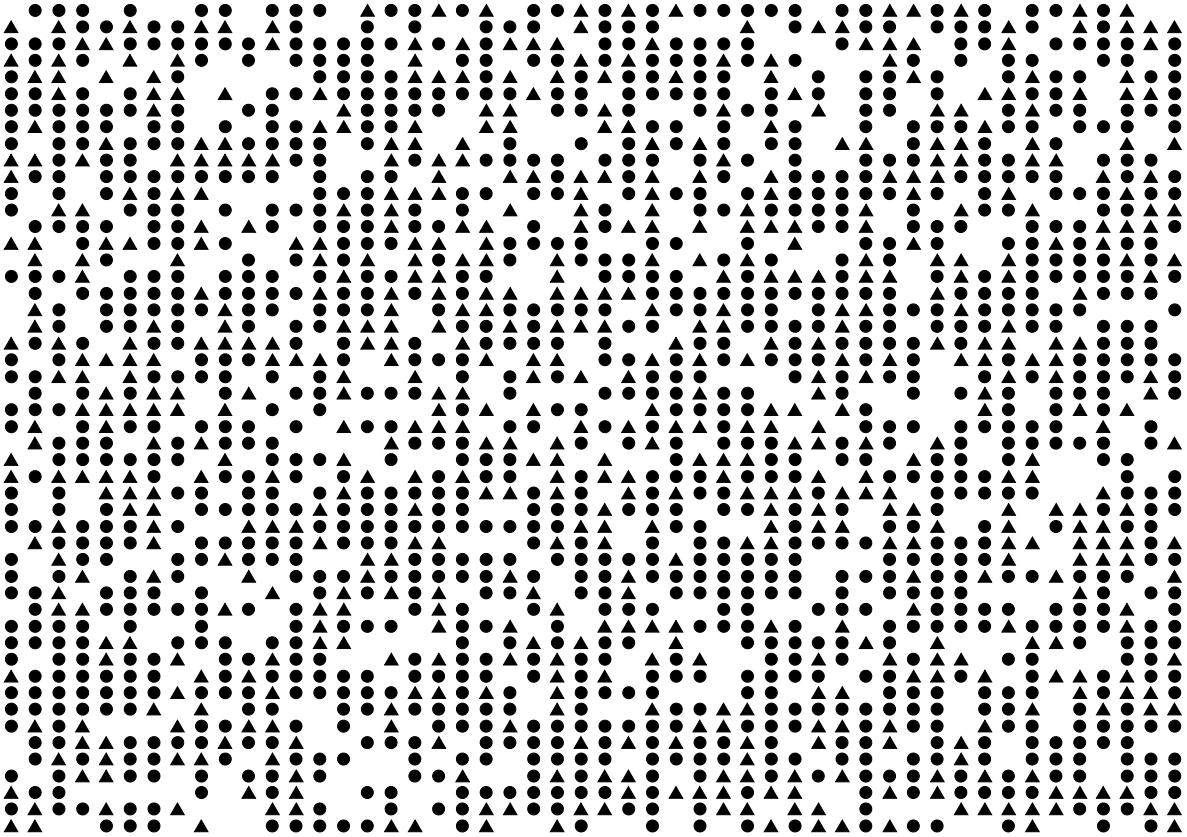
So the proposition is true, working together will deliver much more land to dig for gold.

c for example: 300 metres of rope
4 x 75 metres → area = 75 · 75 = 5625 m²
So how longer the rope is, how bigger the land will be.
400 metres of rope (4 people working together)
4 x 100 metres → area = 100 · 100 = 10000 m²

Progression steps for Golden Rectangles

Progress ⇓	Representing	Analysing	Interpreting and evaluating	Communicating
	The student draws one or two rectangles with a perimeter of 100m.	The student works out the areas of their rectangles correctly.	The student draws several rectangles but not a square and the justification is incorrect or omitted.	The work is communicated adequately, but there are gaps and/or omissions.
	Draws several rectangles.	Calculates the areas of their rectangles and attempts to come to some generalisation.	Realises that different shapes have different areas but comes to incorrect or incomplete conclusion.	The work is communicated clearly and the reasoning may be followed.
	Draws several, correct rectangles for an adventurer working alone and for 2 working together. May draw far too many rectangles.	Calculates the areas correctly and finds that a square is best for 1 adventurer and that 2 working together do better than alone.	Attempts to give some explanation for their findings.	The work is communicated clearly and the reasoning may be easily followed.
	Draws an appropriate number of rectangles and collects the data in an organised way.	Calculates the correct areas, finds that a square is best for 1 adventurer and that 2 working together do better than alone. Finds a rule or pattern in their results.	Gives reasoned explanations for their findings.	Explains work clearly and may consider other shapes.

Handout 3: Counting Trees



This diagram shows some trees in a plantation.

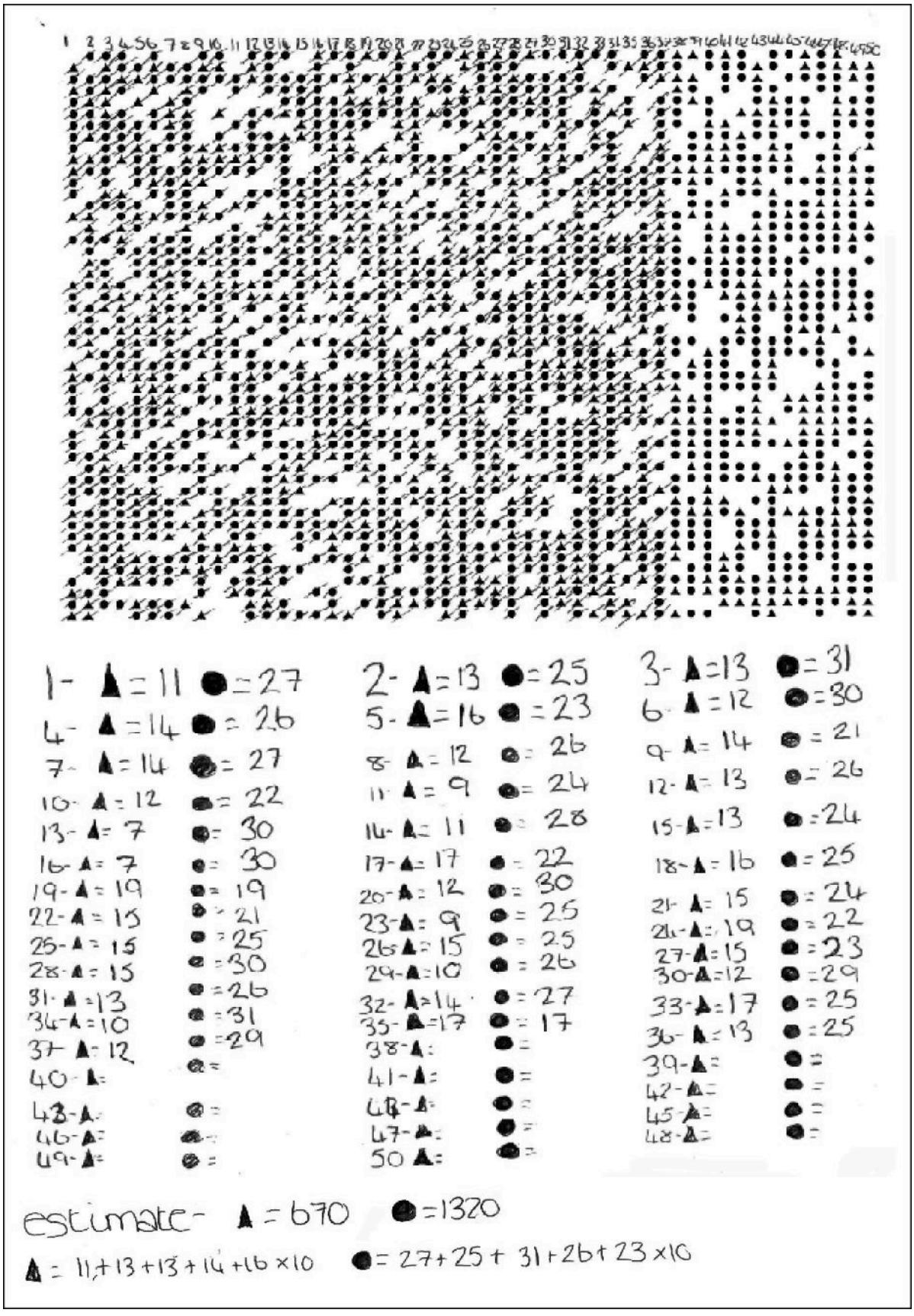
The circles ● show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

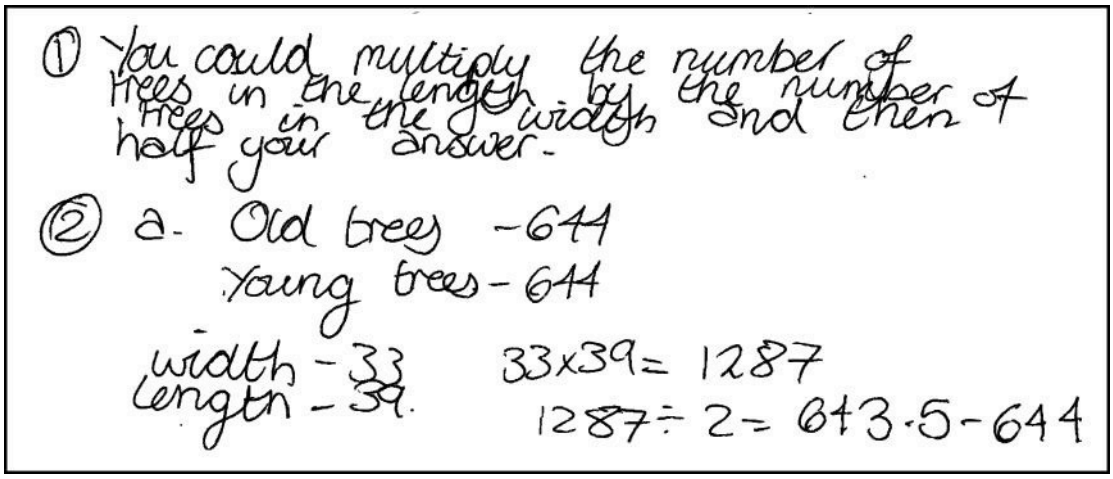
1. What method could he use to estimate the number of trees of each type? Explain your method fully.

2. On your worksheet, use your method to estimate the number of:
    a. Old trees
    b. Young trees

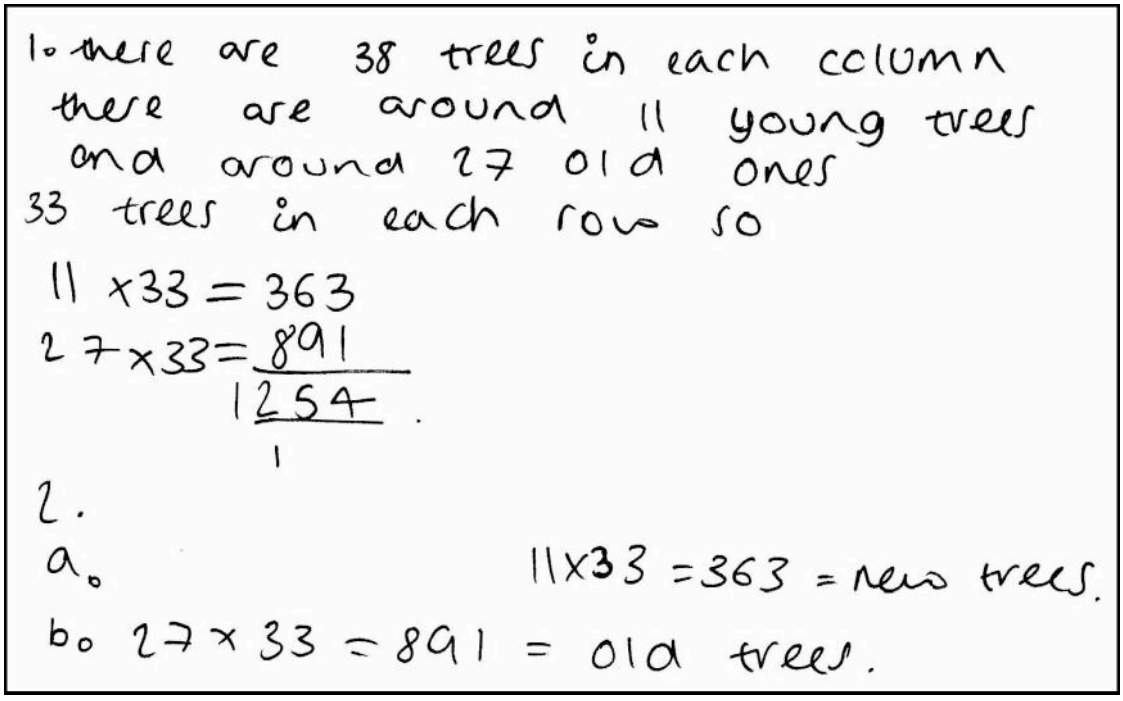
Sample response: Sarah



Sample response: Laura



Sample response: Jenny



Sample response: Woody

2 columns has 21 young trees
55 old

50 columns is approx
50 ÷ 2 = 25
25 × 21 = amount of young trees = 525
25 × 55 = amount of old trees = 1,375
rounded up

young 530
old 1,380

Sample response: Amber

Counting trees

1. If Tom draws a 10X10 square round some trees and counts how many old and new there are. There are 50 rows and 50 columns altogether so he must multiply by 25. He could do this a few times to check and then take the average.

2.

53 old	x 25	= 1325	old
28 new	x 25	= 700	new
19 spaces	x 25	= 475	spaces
100		2500	

1325 + 1200 ÷ 2 = 1262.5
700 + 875 ÷ 2 = 787.5

Check

48 old	x 25	= 1200	old
35 new	x 25	= 875	new
17 spaes	x 25	= 425	spaes
100		2500	

So about 1263 old trees and 788 new trees

Progression steps for Counting Trees

	Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
⇓⇓⇓ Progress ⇓⇓⇓	Chooses a method, but this may not involve sampling. E.g. Counts all trees or multiplies the number of trees in a row by the number in a column.	Follows chosen method, possibly making errors. E.g. Does not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due to method and errors.	Communicates work adequately but with omissions.
	Chooses a sampling method but this is unrepresentative or too small. E.g. tries to count the trees in first row and multiplies by the number of rows.	Follows chosen method, mostly accurately. E.g. May not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due mainly to the method.	Communicates reasoning and results adequately, but with omissions.
	Chooses a reasonable sampling method.	Follows chosen method, mostly accurately.	Estimates a reasonable number of old and new trees in the plantation. The reasonableness of the estimate is not checked. E.g. by repeating with a different sample.	Explains what they are doing but explanation may lack detail.
	Chooses an appropriate sampling technique.	Follows chosen method accurately. Uses a proportional argument correctly.	Deduces a reasonable number of old and new trees in the plantation. There is some evidence of checking the estimate. E.g. Considers a different sampling method.	Communicates reasoning clearly and fully.

Handout 4: Comments on Emma's and Shane's lessons

Answer the following questions after watching the video.

What observations do pupils make about each other's work and how might this help them to improve their own work?

Comment on Emma's and Shane's 'progression steps' in terms of how they might help students to reflect on and improve their work?

Handout 5: Track A: Planning a lesson using students' own work

In this lesson the students work in pairs or threes. Give each group a large sheet of card and a felt-tipped pen. Each group will be working with their initial attempts at the problem.

Explain to the students that you want them to:

- look again at their answers;
- take it in turns to describe their individual attempts to the rest of the group;
- make suggestions after each individual explanation of what they like about the method and also where it might be improved;
- work together as a group to produce a better answer than they did separately;
- make a poster showing their best ideas and the thinking behind these.

Remember to use the students' group work as an opportunity for eliciting evidence of their understanding. As the students are working, you can walk round the room, listening, assessing their reasoning and making appropriate interventions. Listen specifically to pupils that struggled with the task when they worked alone, and offer them support. Have some pre-planned extension activities ready for pupils who have correctly completed the task. You could also ask them to explain their thinking to other students.

Ask pupils to exchange their posters with another group and, on a separate sheet of paper, write comments on:

- Representing - did they choose a good method?
- Analysing - is the reasoning correct and are the calculations accurate?
- Interpreting - are the conclusions sensible?
- Communication - was the reasoning easy to understand and follow?

As they do this, go around encouraging pupils to read the work carefully and comment on the points mentioned. When pupils have commented on the work, one person from the group should take the poster to the group that produced it, and explain what needs to be done for the work to be improved further.

Give groups a little time to absorb the comments and time to further improve their ideas. Towards the end of the lesson hold a discussion on the approaches used and the changes that have been made.

Ask the students:

- What changes have you made to your initial work?
- Why is it better now than it was before?

Collect in the work and use this for your own reflection on how student thinking has changed or improved through these peer and self-assessment processes.

Handout 6: Track B: Planning a lesson using sample work provided by the teacher

This approach uses samples of work taken from another class that the students assess. Provide the students, working individually or in small groups, with the samples and outline what you want them to do.

Explain to the students that you want them to:

- imagine they are the teacher;
- look at the sample work carefully;
- try to find any new ideas they had not thought of themselves;
- try to find the mistakes.

Students should then be asked to comment on each of the following themes:

- Representing - did they choose a good method?
- Analysing - is the reasoning correct and are the calculations accurate?
- Interpreting - are the conclusions sensible?
- Communication - was the reasoning easy to understand and follow?

In this way, pupils will become more aware of what is valued in their work. Whilst they are working, listen to their discussions and encourage them to think more deeply. Encourage them to say what they like and dislike about each response and ask them to explain their reasons.

For a whole class discussion then project each piece of sample student work on the board and ask the students to make comments.

Ask the students to share some of the comments they wrote down and answer the following questions:

- What did you think about this piece of work?
- What did you think of the methods they chose?
- Which method did you like best? Why was this?
- Did you find any mistakes in their work?
- Do you agree with their conclusions?

Then ask the students to work together and improve their own solutions, using what they have learned. As they do this, ask pupils to explain their thinking. Try to assess how much pupils have learned from the sharing session.

Handout 7: Planning for an assessment lesson

When will you allow students time to work on the assessment task, individually or in pairs, without your guidance?

Do students have an opportunity to think about their own work, discuss the other students' work and reflect on their thinking and methods?

Do students have sufficient time to revise their own work in the light of the comments and discussion?

What materials do students need for this lesson?

How will you gain insight of your students' thinking during their work in the lesson?

What difficulties do you expect from your students? How can you react to these?

Handout 8: Exchanging experiences

How did your students perform on the task, unaided?

What were their strengths and difficulties?

How did students assess the provided responses or work of their peers? What aspects did they attend to? How did they formulate feedback?

How well did students react to and use the evidence to improve their own work?

What are the implications of this lesson for your future teaching?