

STUDENTS BECOMING ASSESSORS

HANDOUTS FOR TEACHERS

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Handout 1: Text messaging



1. How many text messages are sent if four people all send messages to each other?
2. How many text messages are sent with different numbers of people?
3. Approximately how many text messages would travel in cyberspace if everyone in your school took part?
4. Can you think of other situations that would give rise to the same mathematical relationship?

This was adapted from Sending texts – a task from the Nuffield Foundation's Applying Mathematical Processes project – see <http://www.nuffieldcurriculumcentre.org/>

Follow-up task for students

Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

Tom's answer

Celia send's one to Tracey = 1
 Tracey send's one to Celia = 1
 Tracey send's one to Maria = 1
 Maria send's one to Anne - Maria = 1
 Anne - marie send's one to Celia = 1
 Celia send's one to Anne - Marie = 1
 Maria send's one to Tracey = 1
 Tracey send's one to Anne marie = 1
 Maria send's one to Celia = 1

Sam's answer

- ① For 4 people  12 .
- ② 1)  0. 2)  1 2 3)  11 11 11 6. 4)  11 11 11 11 12
- 5)  11  11  11  11 16. 6)  11  11  11  11 20
- 7)  11  11  11  11  11  11  11 42
- 8)  11  11  11  11  11  11  11  11 56
- 9)  11  11  11  11  11  11  11  11 73
- ③ Don't know.

Chris's answer

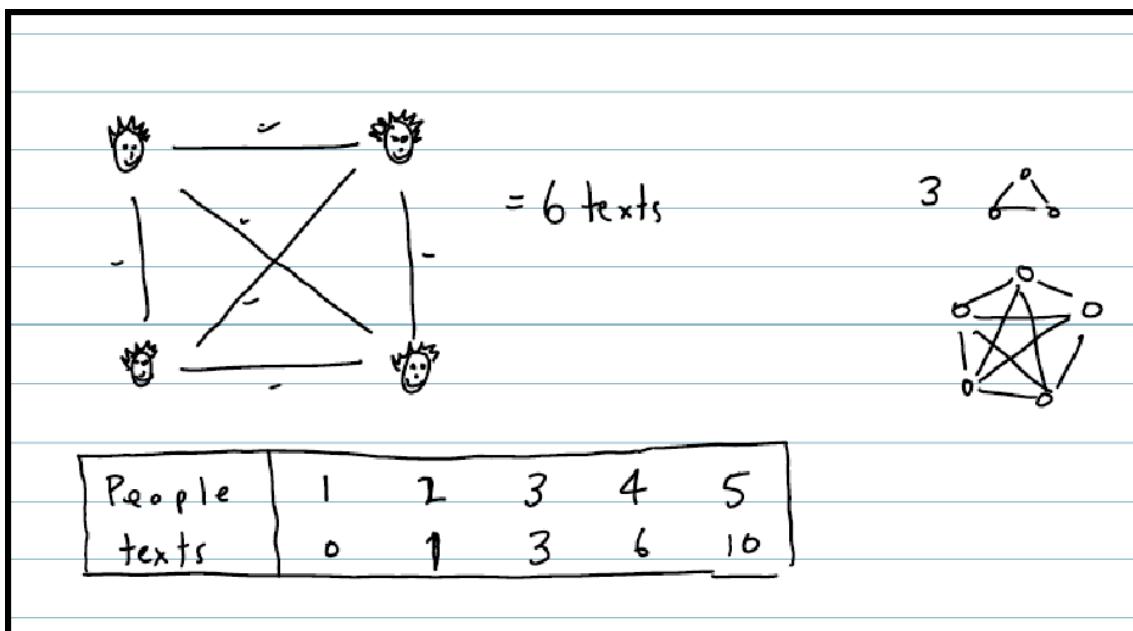


Diagram showing 4 people in a square. Every person is connected to every other person by a line, forming a complete graph. Below the diagram is a table showing the number of texts sent as a function of the number of people.

People	1	2	3	4	5
texts	0	1	3	6	10

Lily's answer

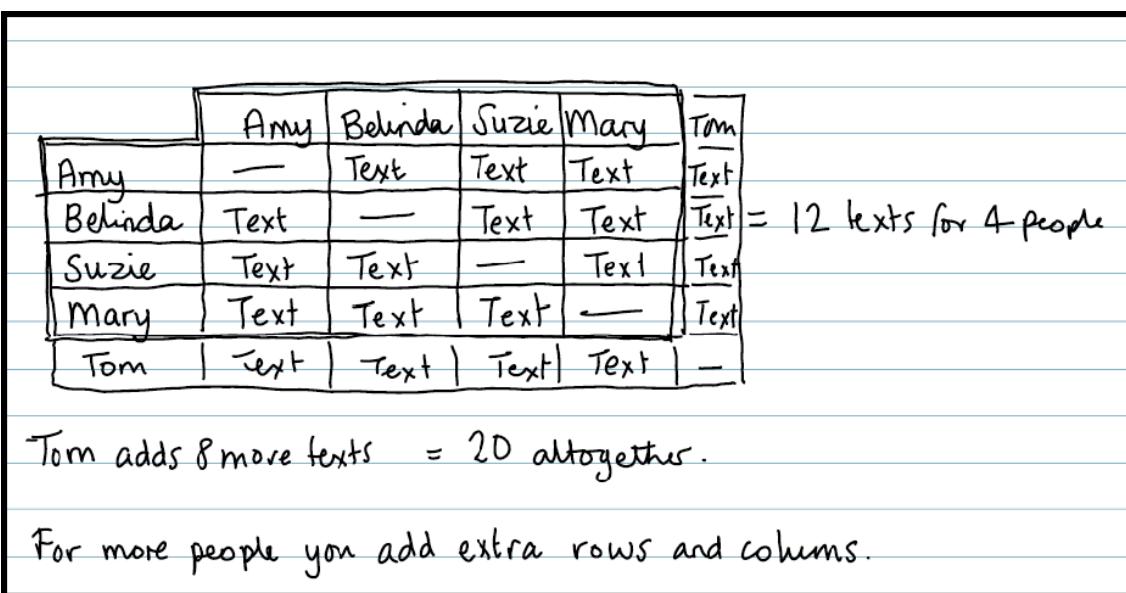
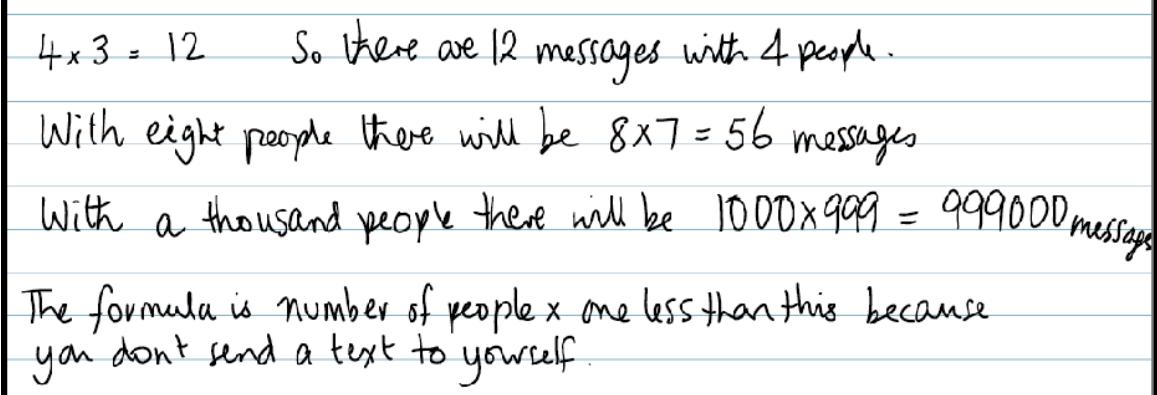


Diagram showing a 4x5 matrix for 4 people (Amy, Belinda, Suzie, Mary) and 1 additional person (Tom). The matrix shows that each person sends a text to the other 4 people. Below the matrix, it is noted that Tom adds 8 more texts, making a total of 20. A note says "For more people you add extra rows and columns."

Marvin's answer



Handwritten calculations by Marvin:

$$4 \times 3 = 12 \quad \text{So there are 12 messages with 4 people.}$$

With eight people there will be $8 \times 7 = 56$ messages

With a thousand people there will be $1000 \times 999 = 999000$ messages

The formula is number of people x one less than this because you don't send a text to yourself.

Reflecting on Sheena's lesson

What aspects of the provided samples of work do students attend to?

What criteria do students use as they assess the sample work?

What are students learning from the sample work?

Concerns about using student work

Comment on Sheena's orchestration of the activity.

What could you do to avoid potential problems when your students assess other students' work?

Choose a topic or lesson that you have recently taught or planned and think about the samples you might use.

Briefly outline the lesson.

What specific examples would be useful?

Why would they be useful?

What criteria would you use for choosing sample work to use with students?

How would you introduce the activity in your class?

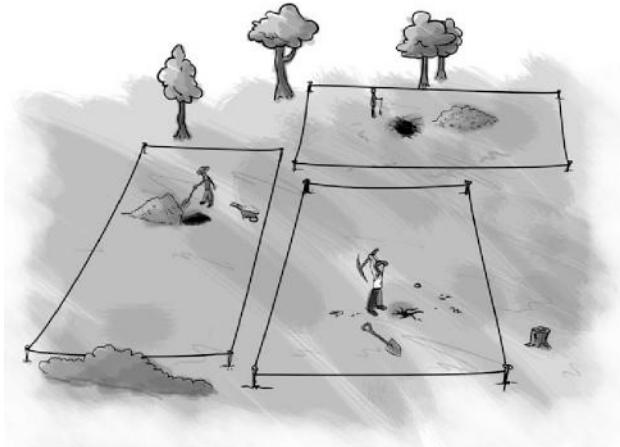
What would you be looking out for as you circulate, in the time that the students are discussing the samples? Why?

Handout 2: Golden Rectangles

In the 19th century, many adventurers travelled to North America to search for gold.

A man named Dan Jackson owned some land where gold had been found.

Instead of digging for the gold himself, he rented plots of land to the adventurers.



Dan gave each adventurer four wooden stakes and a rope measuring exactly 100 metres.

Each adventurer had to use the stakes and the rope to mark off a rectangular plot of land.

- Assuming each adventurer would like to have the biggest plot, how should he place his stakes?

Explain your answer.

- Read the following proposition:

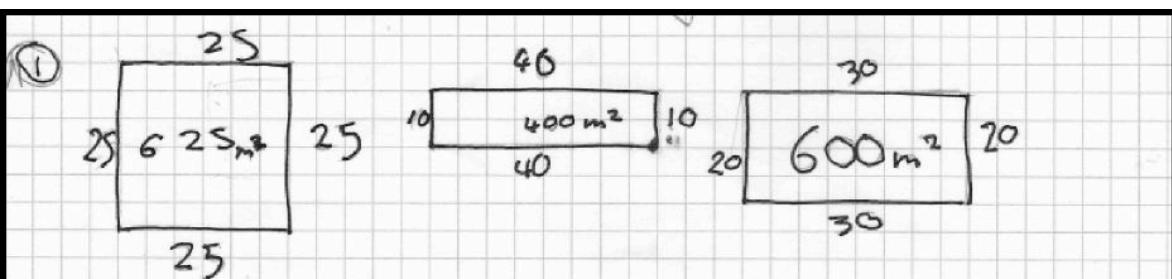
"Tie the ropes together! You can get more land if you work together than if you work separately."

Investigate whether the proposition is true for two adventurers working, still using four stakes.

- Is the proposition true for more than two people? Explain your answer.

Alvin's answer

①



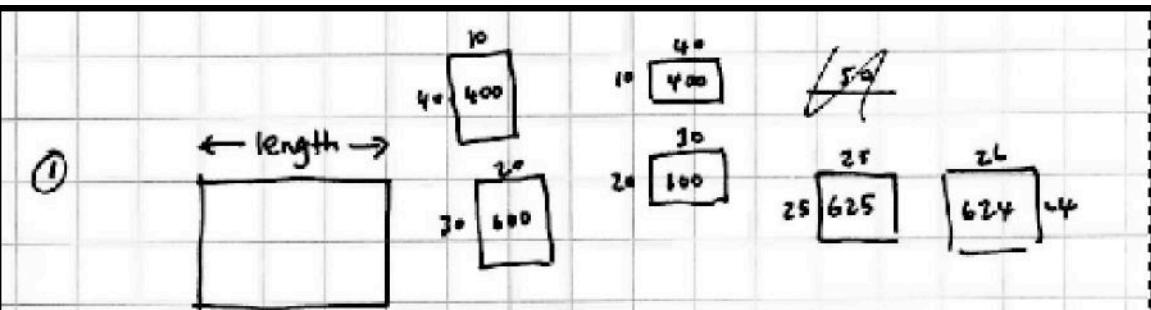
IF you want the biggest plot,
I think you need the biggest area,
so what I did was draw the rectangles
out and I found out that the
more equal it is the bigger the area.

② IT IS better to work on your
own because if you work together
there will be a bigger area but
you will have to half it with
the other person, For example, If
you combine the ropes you will have
250m, If you do 50×50 to find
the area it will be $2500m^2$ but you
will need to half that with other person
so that will give you $1250m^2$, so
you will have more to do. so
it is easier to work on your own.

③ NO it is not true for more
than 2 people, they will have to
work harder.

Bernie's answer

①

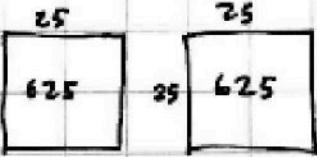


I will change the length and see how the area changes.

length	10	20	30	40	50	25	26	
Area	400	800	600	400	X	625	624	

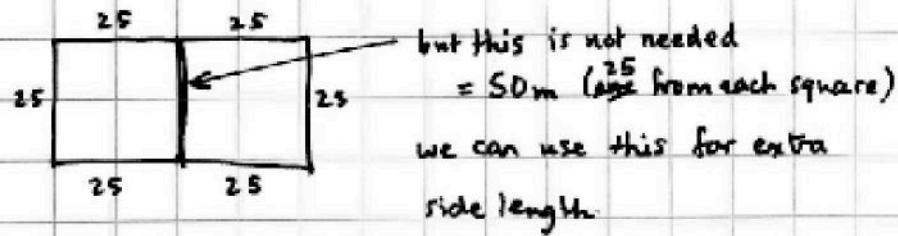
So a length of 25 is best.

② If two people work apart they get



$$= 1250 \text{ m}^2$$

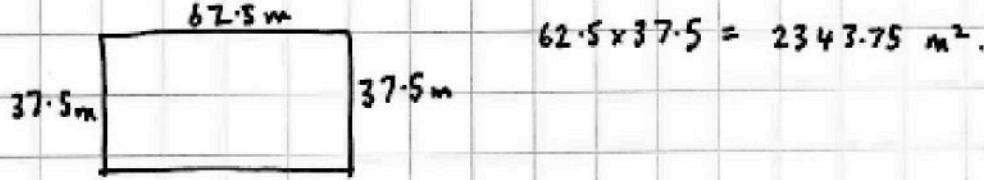
If they work together they get



we can use this for extra side length.

$50 \text{ m} \div 4 = 12.5 \text{ m}$.

Add 12.5m onto each side:

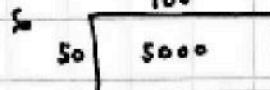


$$62.5 \times 37.5 = 2343.75 \text{ m}^2.$$

③ If 3 people



$= 100 \text{ m}$ not needed.
 $\div 4 = 25$



Chris's answer

a $25 \times 25 = 625 \text{ m}^2$

~~30 x 20 = 600 m²~~

$30 \times 20 = 600 \text{ m}^2$

$40 \times 10 = 400 \text{ m}^2$

He should place the stakes in a ~~square~~ ^{rectangle}, because then he has the most land. But the rectangle need to be $30 \times 20 \text{ m}$.

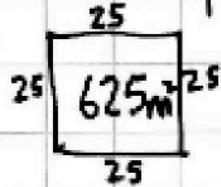
b With two ropes of 100 m, you can get a bigger amount of land. If you take $55 \text{ m} \times 45 \text{ m}$, you get more than the double amount of land. $55 \times 45 = 2475$, $2475 \text{ m}^2 : 2 = 1237.5 \text{ m}^2$

¶

c Yes, because you can make the plot of land bigger in that way everyone has more land.
If the plot of land is 80×70 , the land is 5600 m^2 .
 $5600 \text{ m}^2 : 3 = 1866.67 \rightarrow 1866.7 \text{ m}^2$ per person.
That is more land.

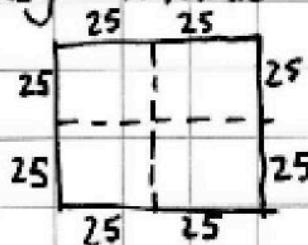
Danny's answer

① He should place his stakes in a square to give the biggest area like this



② If two adventurers work together they will have 200m² of rope so they can make a square twice as long and wide.

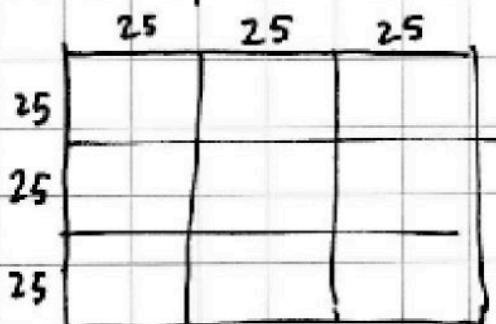
= 4 x area.



This is much better than 2x area.

③ If three work together they will have 300m² of rope so they can make a square three times as long and wide

= 9 x area



This is much better than 3x area.

I think that the area goes up by square numbers each time.

Elsie's answer

a. 4×25 metres \rightarrow area = $25 \times 25 = 625 \text{ m}^2$

2×20 & $2 \times 30 \rightarrow$ area = $20 \times 30 = 600 \text{ m}^2$

2×10 & $2 \times 40 \rightarrow$ area = $10 \times 40 = 400 \text{ m}^2$

So 4×25 metres would make the biggest area.

b. 2×100 metres of rope = 200 m .

4×50 metres \rightarrow area = $50 \times 50 = 2500 \text{ m}^2$

2×20 & $2 \times 80 \rightarrow$ area = $20 \times 80 = 1600 \text{ m}^2$

2×30 & $2 \times 70 \rightarrow$ area = $30 \times 70 = 2100 \text{ m}^2$

2×40 & $2 \times 60 \rightarrow$ area = $40 \times 60 = 2400 \text{ m}^2$

2×10 & $2 \times 90 \rightarrow$ area = $10 \times 90 = 900 \text{ m}^2$

So the proposition is true, working together will deliver much more land to dig for gold.

c. for example: 300 metres of ~~8~~ rope

4×75 metres \rightarrow area = $75 \times 75 = 5625 \text{ m}^2$

So how longer the rope is, how bigger the land will be.

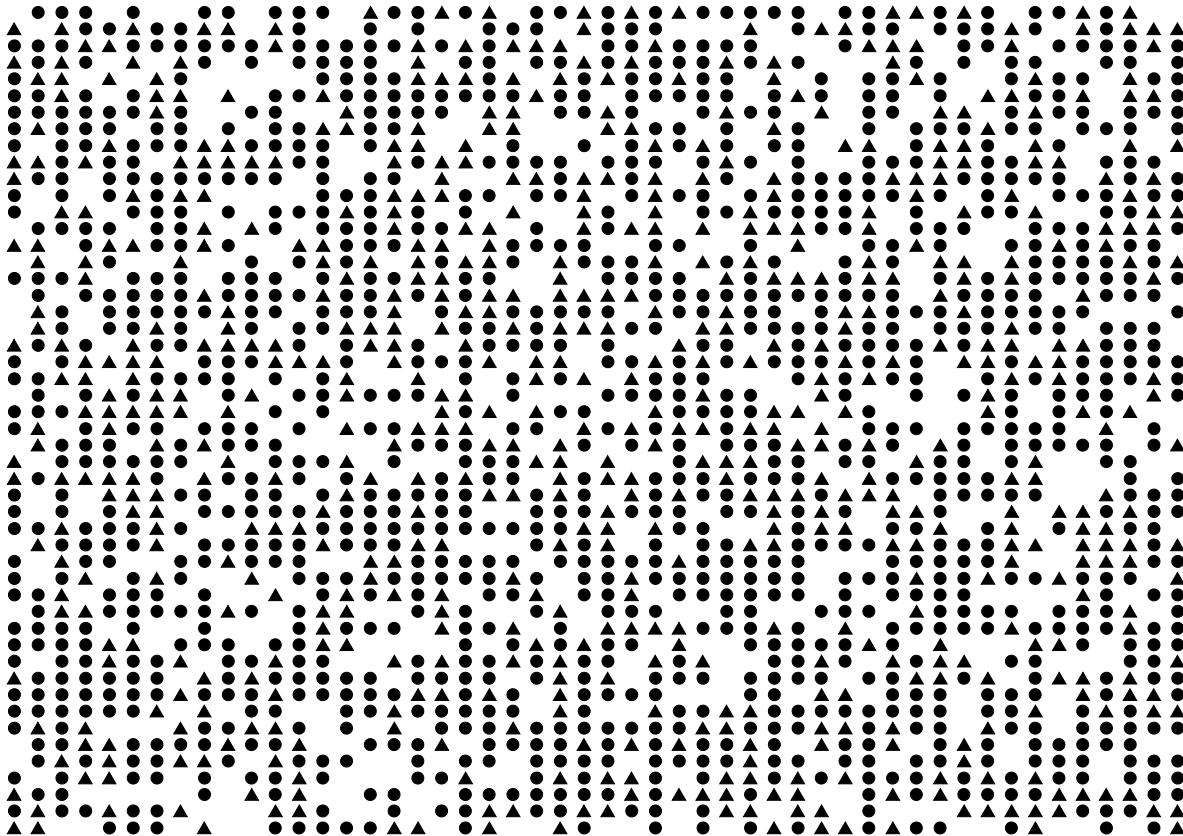
400 metres of rope (4 people working together)

4×100 metres \rightarrow area = $100 \times 100 = 10000 \text{ m}^2$

Progression steps for Golden Rectangles

	Representing	Analysing	Interpreting and evaluating	Communicating
↓ ↓ ↓ Progress ↓ ↓ ↓	<p>The student draws one or two rectangles with a perimeter of 100m.</p> <p>Draws several rectangles.</p> <p>Draws several, correct rectangles for an adventurer working alone and for 2 working together. May draw far too many rectangles.</p> <p>Draws an appropriate number of rectangles and collects the data in an organised way.</p>	<p>The student works out the areas of their rectangles correctly.</p> <p>Calculates the areas of their rectangles and attempts to come to some generalisation.</p> <p>Calculates the areas correctly and finds that a square is best for 1 adventurer and that 2 working together do better than alone.</p> <p>Calculates the correct areas, finds that a square is best for 1 adventurer and that 2 working together do better than alone.</p> <p>Finds a rule or pattern in their results.</p>	<p>The student draws several rectangles but not a square and the justification is incorrect or omitted.</p> <p>Realises that different shapes have different areas but comes to incorrect or incomplete conclusion.</p> <p>Attempts to give some explanation for their findings.</p> <p>Gives reasoned explanations for their findings.</p>	<p>The work is communicated adequately, but there are gaps and/or omissions.</p> <p>The work is communicated clearly and the reasoning may be followed.</p> <p>The work is communicated clearly and the reasoning may be easily followed.</p> <p>Explains work clearly and may consider other shapes.</p>

Handout 3: Counting Trees



This diagram shows some trees in a plantation.

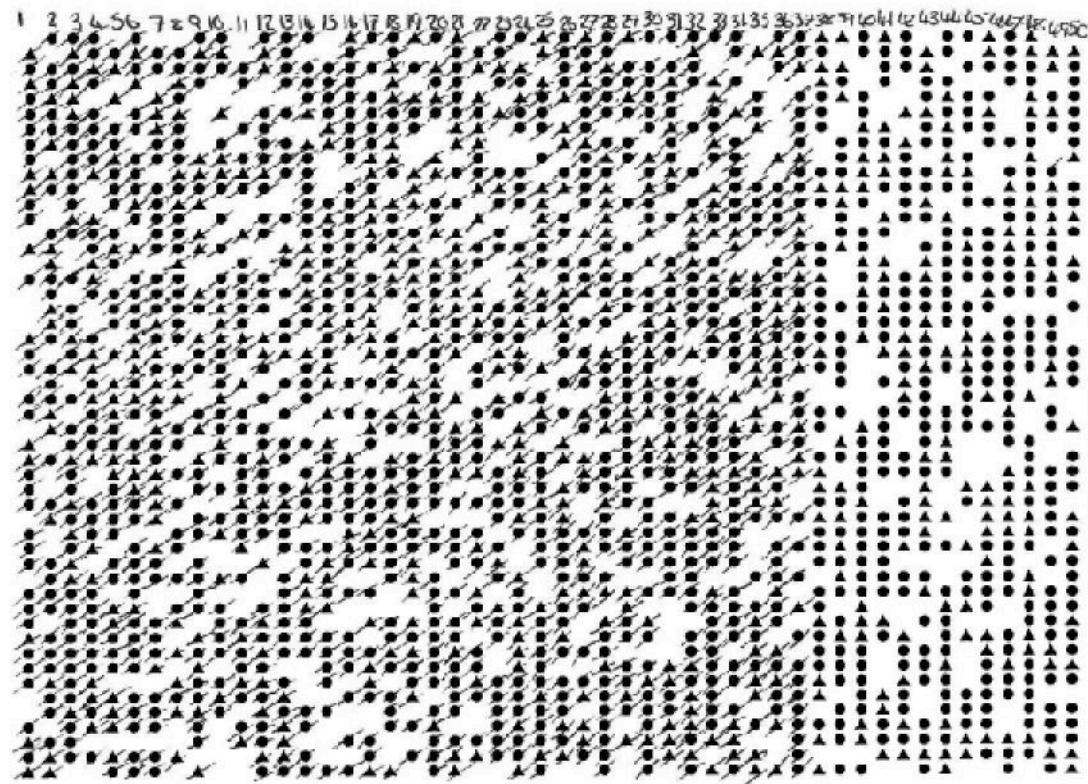
The circles ● show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type? Explain your method fully.

2. On your worksheet, use your method to estimate the number of:
 - a. Old trees
 - b. Young trees

Sample response: Sarah



$$1 - \blacktriangle = 11 \quad \bullet = 27$$

$$4 - \blacktriangle = 14 \quad \bullet = 26$$

$$7 - \blacktriangle = 14 \quad \bullet = 27$$

$$10 - \blacktriangle = 12 \quad \bullet = 22$$

$$13 - \blacktriangle = 7 \quad \bullet = 30$$

$$16 - \blacktriangle = 7 \quad \bullet = 30$$

$$19 - \blacktriangle = 19 \quad \bullet = 19$$

$$22 - \blacktriangle = 15 \quad \bullet = 21$$

$$25 - \blacktriangle = 15 \quad \bullet = 25$$

$$28 - \blacktriangle = 15 \quad \bullet = 30$$

$$31 - \blacktriangle = 13 \quad \bullet = 26$$

$$34 - \blacktriangle = 10 \quad \bullet = 31$$

$$37 - \blacktriangle = 12 \quad \bullet = 29$$

$$40 - \blacktriangle = \quad \bullet =$$

$$43 - \blacktriangle = \quad \bullet =$$

$$46 - \blacktriangle = \quad \bullet =$$

$$49 - \blacktriangle = \quad \bullet =$$

$$2 - \blacktriangle = 13 \quad \bullet = 25$$

$$5 - \blacktriangle = 16 \quad \bullet = 23$$

$$8 - \blacktriangle = 12 \quad \bullet = 26$$

$$11 - \blacktriangle = 9 \quad \bullet = 24$$

$$14 - \blacktriangle = 11 \quad \bullet = 28$$

$$17 - \blacktriangle = 17 \quad \bullet = 22$$

$$20 - \blacktriangle = 12 \quad \bullet = 30$$

$$23 - \blacktriangle = 9 \quad \bullet = 25$$

$$26 - \blacktriangle = 15 \quad \bullet = 25$$

$$29 - \blacktriangle = 10 \quad \bullet = 26$$

$$32 - \blacktriangle = 14 \quad \bullet = 27$$

$$35 - \blacktriangle = 17 \quad \bullet = 17$$

$$38 - \blacktriangle = \quad \bullet =$$

$$41 - \blacktriangle = \quad \bullet =$$

$$44 - \blacktriangle = \quad \bullet =$$

$$47 - \blacktriangle = \quad \bullet =$$

$$50 - \blacktriangle = \quad \bullet =$$

$$3 - \blacktriangle = 13 \quad \bullet = 31$$

$$6 - \blacktriangle = 12 \quad \bullet = 30$$

$$9 - \blacktriangle = 14 \quad \bullet = 21$$

$$12 - \blacktriangle = 13 \quad \bullet = 26$$

$$15 - \blacktriangle = 13 \quad \bullet = 24$$

$$18 - \blacktriangle = 16 \quad \bullet = 25$$

$$21 - \blacktriangle = 15 \quad \bullet = 24$$

$$24 - \blacktriangle = 19 \quad \bullet = 22$$

$$27 - \blacktriangle = 15 \quad \bullet = 23$$

$$30 - \blacktriangle = 12 \quad \bullet = 29$$

$$33 - \blacktriangle = 17 \quad \bullet = 25$$

$$36 - \blacktriangle = 13 \quad \bullet = 25$$

$$39 - \blacktriangle = \quad \bullet =$$

$$42 - \blacktriangle = \quad \bullet =$$

$$45 - \blacktriangle = \quad \bullet =$$

$$48 - \blacktriangle = \quad \bullet =$$

estimate - $\blacktriangle = 670 \quad \bullet = 1320$

$$\blacktriangle = 11 + 13 + 13 + 14 + 16 \times 10 \quad \bullet = 27 + 25 + 31 + 26 + 23 \times 10$$

Sample response: Laura

① You could multiply the number of trees in the length by the number of trees in the width and then half your answer.

② a. Old trees - 644
 Young trees - 644
 width - 33 $33 \times 39 = 1287$
 length - 39 $1287 \div 2 = 643.5 - 644$

Sample response: Jenny

1. there are 38 trees in each column
 there are around 11 young trees
 and around 27 old ones
 33 trees in each row so

$$11 \times 33 = 363$$

$$27 \times 33 = \underline{891}$$

$$1254$$

2.
 a. $11 \times 33 = 363$ = new trees.
 b. $27 \times 33 = 891$ = old trees.

Sample response: Woody

2 columns has 21 young trees
 55 old

50 columns is approx

$$50 \div 2 = 25$$

$$25 \times 21 = \text{amount of young trees} = 525$$

$$25 \times 55 = \text{amount of old trees} = 1,375$$

rounded up

$$\begin{array}{r} \text{young} \ 530 \\ \text{old} \ 1,380 \end{array}$$

Sample response: Amber

Counting trees

1. If Tom draws a 10X10 square round some trees and counts how many old and new there are. There are 50 rows and 50 columns altogether so he must multiply by 25. He could do this a few times to check and then take the average.

2.

$$\begin{array}{rcl} 53 \text{ old} & \times 25 & = 1325 \text{ old} \\ 28 \text{ new} & \times 25 & = 700 \text{ new} \\ \hline 19 \text{ spaces} & \times 25 & = 475 \text{ spaces} \\ \hline 100 & & 25.00 \end{array}$$

$$1325 + 1200 \div 2 = 1262.5$$

$$700 + 875 \div 2 = 787.5$$

check

$$\begin{array}{rcl} 48 \text{ old} & \times 25 & = 1200 \text{ old} \\ 35 \text{ new} & \times 25 & = 875 \text{ new} \\ \hline 17 \text{ spaces} & \times 25 & = 425 \text{ spaces} \\ \hline 100 & & 25.00 \end{array}$$

So about 1263 old trees
 and 788 new trees

~~1263~~ ~~788~~

Progression steps for Counting Trees

	Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
↓	Chooses a method, but this may not involve sampling. E.g. Counts all trees or multiplies the number of trees in a row by the number in a column.	Follows chosen method, possibly making errors. E.g. Does not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due to method and errors.	Communicates work adequately but with omissions.
↓	Chooses a sampling method but this is unrepresentative or too small. E.g. tries to count the trees in first row and multiplies by the number of rows.	Follows chosen method, mostly accurately. E.g. May not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due mainly to the method.	Communicates reasoning and results adequately, but with omissions.
↓	Chooses a reasonable sampling method.	Follows chosen method, mostly accurately.	Estimates a reasonable number of old and new trees in the plantation. The reasonableness of the estimate is not checked. E.g. by repeating with a different sample.	Explains what they are doing but explanation may lack detail.
↓	Chooses an appropriate sampling technique.	Follows chosen method accurately. Uses a proportional argument correctly.	Deduces a reasonable number of old and new trees in the plantation. There is some evidence of checking the estimate. E.g. Considers a different sampling method.	Communicates reasoning clearly and fully.

Handout 4: Comments on Emma's and Shane's lessons

Answer the following questions after watching the video.

What observations do pupils make about each other's work and how might this help them to improve their own work?

Comment on Emma's and Shane's 'progression steps' in terms of how they might help students to reflect on and improve their work?

Handout 5: Track A: Planning a lesson using students' own work

In this lesson the students work in pairs or threes. Give each group a large sheet of card and a felt-tipped pen. Each group will be working with their initial attempts at the problem.

Explain to the students that you want them to:

- look again at their answers;
- take it in turns to describe their individual attempts to the rest of the group;
- make suggestions after each individual explanation of what they like about the method and also where it might be improved;
- work together as a group to produce a better answer than they did separately;
- make a poster showing their best ideas and the thinking behind these.

Remember to use the students' group work as an opportunity for eliciting evidence of their understanding. As the students are working, you can walk round the room, listening, assessing their reasoning and making appropriate interventions. Listen specifically to pupils that struggled with the task when they worked alone, and offer them support. Have some pre-planned extension activities ready for pupils who have correctly completed the task. You could also ask them to explain their thinking to other students.

Ask pupils to exchange their posters with another group and, on a separate sheet of paper, write comments on:

- Representing - did they choose a good method?
- Analysing - is the reasoning correct and are the calculations accurate?
- Interpreting - are the conclusions sensible?
- Communication - was the reasoning easy to understand and follow?

As they do this, go around encouraging pupils to read the work carefully and comment on the points mentioned. When pupils have commented on the work, one person from the group should take the poster to the group that produced it, and explain what needs to be done for the work to be improved further.

Give groups a little time to absorb the comments and time to further improve their ideas. Towards the end of the lesson hold a discussion on the approaches used and the changes that have been made.

Ask the students:

- What changes have you made to your initial work?
- Why is it better now than it was before?

Collect in the work and use this for your own reflection on how student thinking has changed or improved through these peer and self-assessment processes.

Handout 6: Track B: Planning a lesson using sample work provided by the teacher

This approach uses samples of work taken from another class that the students assess. Provide the students, working individually or in small groups, with the samples and outline what you want them to do.

Explain to the students that you want them to:

- imagine they are the teacher;
- look at the sample work carefully;
- try to find any new ideas they had not thought of themselves;
- try to find the mistakes.

Students should then be asked to comment on each of the following themes:

- Representing - did they choose a good method?
- Analysing - is the reasoning correct and are the calculations accurate?
- Interpreting - are the conclusions sensible?
- Communication - was the reasoning easy to understand and follow?

In this way, pupils will become more aware of what is valued in their work. Whilst they are working, listen to their discussions and encourage them to think more deeply. Encourage them to say what they like and dislike about each response and ask them to explain their reasons.

For a whole class discussion then project each piece of sample student work on the board and ask the students to make comments.

Ask the students to share some of the comments they wrote down and answer the following questions:

- What did you think about this piece of work?
- What did you think of the methods they chose?
- Which method did you like best? Why was this?
- Did you find any mistakes in their work?
- Do you agree with their conclusions?

Then ask the students to work together and improve their own solutions, using what they have learned. As they do this, ask pupils to explain their thinking. Try to assess how much pupils have learned from the sharing session.

Handout 7: Planning for an assessment lesson

When will you allow students time to work on the assessment task, individually or in pairs, without your guidance?

Do students have an opportunity to think about their own work, discuss the other students' work and reflect on their thinking and methods?

Do students have sufficient time to revise their own work in the light of the comments and discussion?

What materials do students need for this lesson?

How will you gain insight of your students' thinking during their work in the lesson?

What difficulties do you expect from your students? How can you react to these?

Handout 8: Exchanging experiences

How did your students perform on the task, unaided?

What were their strengths and difficulties?

How did students assess the provided responses or work of their peers? What aspects did they attend to? How did they formulate feedback?

How well did students react to and use the evidence to improve their own work?

What are the implications of this lesson for your future teaching?