FaSMEd

Raising Achievement through Formative Assessment in Science and Mathematics Education

Applying properties of exponents

Subject: Mathematics

Age of students: 14 - 15 years

Technology: Card

Functionalities: Sending and displaying

Time: 1 or 2 lessons

FaSMEd partner: African Institute for Mathematical Sciences

Schools Enrichment Centre

Short Abstract:

This lesson is about using and applying the laws of exponents. After a short introduction, during which the teacher leads a discussion about powers of 2, the students work in small groups. They are given two sets of small cards to put into groups. On one set expressions, such as $6^8 \div 6^4$, are given. On the other set single values, such as 6^4, are given. Students are required to put cards with equivalent value into groups. After the activity the teacher holds a class discussion aiming to draw out something about the students' own learning and to think about the bigger conceptual issues such as why exponential rules work and if students can find different ways to explain why they work.

1. Content

The lesson is about using and applying properties of exponents. The students need to recognise when a law or property applies, and then to apply the appropriate law accurately. When no law applies they need to find another way to evaluate the expression.

2. Activity

2.1 Aims

This lesson is about embedding and reinforcing knowledge about the rules of exponents, and in particular aims to help students:

- Recall and use the properties of exponents to generate equivalent numeric expressions.
- Identify the appropriate property to use and apply it correctly.
- Check the numerical value of an expression involving exponents without using a calculator.

2.2 Structure / Methodology

Time needed: 60 to 90 minutes

Materials needed:

- Powers of 2 slide (or equivalent) see Figure 1 below
- Mini whiteboards, wipes and pens
- Poster paper and glue
- Sets of small cards for students to match (see Figure 2)
- Sets of big cards for use in the whole-class discussion

2.3 Lesson plan:

This is an abbreviated lesson plan, adapted from the one found at http://map.mathshell.org/lessons.php?unit=8110&collection=8.

Introduction

Remind the learners that exponential notation is a short way of writing repeated multiplication: $2^3 = 2\times2\times2$. Revise terminology if necessary.

Hand out a mini white board (or a blank piece of paper) to each learner (or pair of learners). Write or display the equations in Figure 1 on the board and ask the learners to rewrite each equation on their mini white board or piece of paper so that all the terms are in exponential form.

Powers of 2

A: $8 \times 4 = 32$

B: $16 \div 8 = 2$

C: $8 \div 16 = \frac{1}{2}$

D: $8 \div 8 = 1$

Figure 1: Number sentences to write as powers of 2

Ask the learners to show you their answers. Discuss each equation. Emphasise the fact that an expression in exponential form has a numerical value.

Card matching activity

Put the learners into pairs. Each group should have:

- a set of expression cards (E1-E14, see Figure 2)
- a set of solution cards (S1-S10, see Figure 2)
- a large sheet of paper
- glue or Prestik/bluetack.

E1 $2^2 \times 3^2$	E2 $3^2 - 2^3$
E3 $2^2 + 2^3$	E4 $2^2 \div 2^3$
E5 $6^8 \div 6^4$	E6 $2^2 - 2^2$
E7 $3^2 + 3^3$	E8 $4^2 \div 2^3$
E9 $2^3 \div 2^{-2}$	E10 $(2^3)^2$
E11 3×2^2	E12 $2^3 \times 2^3$
E13 $5^2 - 3^3$	E14 $(3^2 \times 2^2)^2$

S1 2^1	S2 2^5
S3 $(-2)^1$	S4 2^{-1}
S5 2^0	S6 2^6
S7 6^4	S8 6^2
S9 0^2	S10 4^3

Figure 2: Cards for the students to match

Explain that they should match the cards that have the same numerical value. The matching is not one-to-one so one expression card can be matched to two solution cards and vice versa. Once they have matched a number of cards they should stick them down together on the poster. There are 10 groups of cards with the same numerical value.

The second slide in the projector resources summarises these instructions (see Figure 3).

Working Together

1. Work out the value of the cards using the laws of exponents or by calculation.
2. Put cards with the same value in groups.
3. Each group should have AT LEAST one card of each colour.
4. Use the blank card where necessary.
5. You should end up with ten groups of cards.

Figure 3: Instructions for pair work

Allow the learners enough time to do the activity.

Try to avoid telling them the answers. Listen to what they are saying and ask them questions. You may need to ask them about when an exponential law can be used or to do some calculations.

Discussion

Have all the big cards on a table at the front of the class. Ask learners to come up and find a pair of matching cards and to stick these on the board. Alternatively hand out the big 'solution' cards to specific learners and ask them to find matching 'expression' cards and to stick them on the board.

2.4 Technology

- Mini whiteboards for use in the introduction and during the lesson.
- Sets of small cards for use by the small groups of students.
- Sets of big cards for use in the whole class discussion.

All these technologies provide opportunities for formative assessment in that they help to make the students' thinking visible.

2.5 Aspects of Formative Assessment

The teachers assess the students' levels of understanding in real-time and act accordingly by asking questions, giving hints and sometimes telling them the answer. They clarify learning intentions, engineer effective discussions, provide feedback and activate students as resources for one another and for themselves. This takes place at all phases of the lesson.

In the main card-matching activity, students work in pairs to match cards of equal value. The intention is that students will take turns to select and match cards and will explain their thinking to the others in their group. Each student should engage with their peer's explanation and take responsibility for the understanding of others in their group.

If the students follow these instructions, it is likely that they will be assessing their peers' levels of understanding and acting accordingly, sometimes explaining their own reasoning, sometimes telling the peers what to do and sometimes asking questions (i.e. peer formative assessment). In this case they would be activating their peers as instructional resources.

3. Further Information

3.1 Background

This lesson is based on the one designed by the Mathematics Assessment Project, which can be found at http://map.mathshell.org/lessons.php?unit=8110&collection=8.

The lesson design includes the use of a pre-lesson assessment task, which teachers can use to gather information about the current levels of students' understanding and their different problem solving approaches. The use of this task is highly recommended but in practice many teachers do not have time to do so. A post-lesson task is also provided.

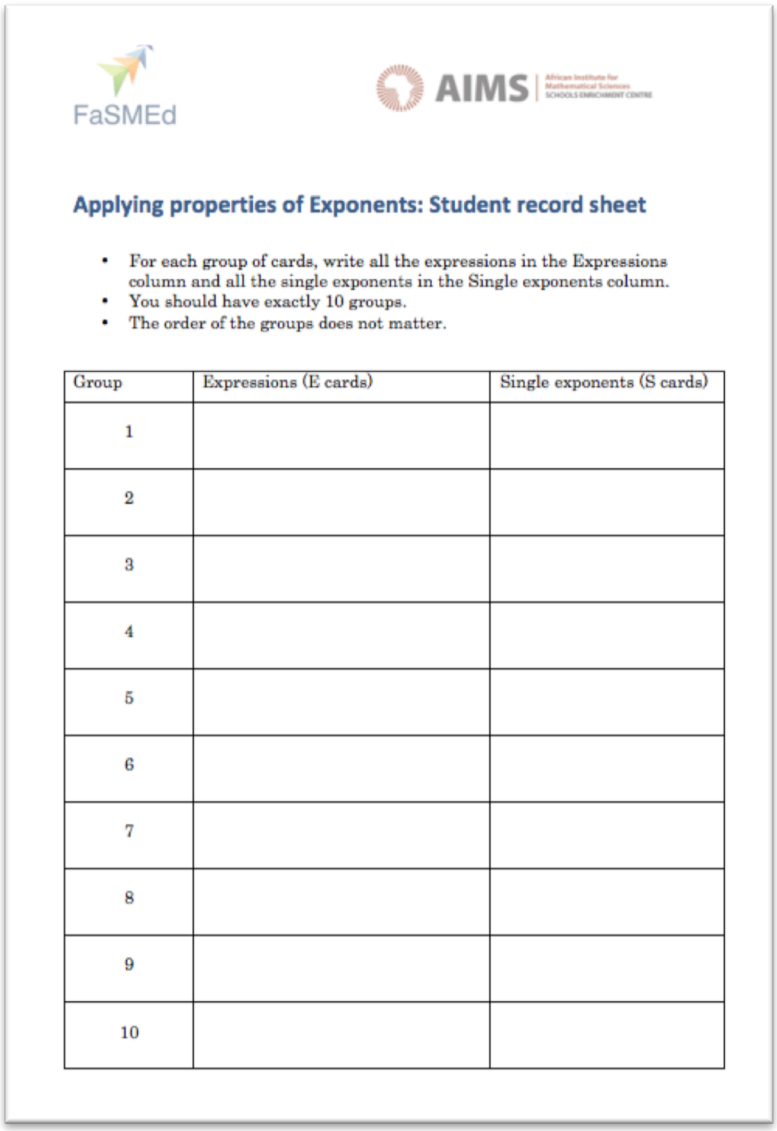
3.2 Common issues

The issues arising in the pre-lesson task are likely to arise in the main card matching activity and the table of common issues below provides some useful questions and prompts teachers could use to move the students' thinking on.

Common issues	Suggested questions and prompts
Student applies the properties $a^m \times a^n = a^{m+n}$ or $a^m \div a^n = a^{m-n}$ incorrectly The student does not appear to understand the conditions under which it is possible to **add** or **subtract** exponents. For example, the student writes: $2+2+2+2 = 2^4$ (Q1a) *or* $2^2 \times 2^3 = 2^6$ (Q1c) or $5^6 \div 5^2 = 5^3$ (Q1g) *or* $5^2 - 3^2 = 2^0$ (Q1h)	• What does the exponent tell you? • What is the difference between 5^2 and 5x2? • Write out $2^3 \times 3^3$ as a product of integers. Now write out 6^6 as a product of integers. Are these equal? • What does 5^6 mean? If I divide this by 5, what is the result? How would you write this as a power of 5? • What is the answer to $2^3 - 2^2$? How would you write this as an integer? How would you write this as a power of 2? Why should you not subtract the powers?
Student applies the property $a^m \times b^m = (ab)^m$ incorrectly The student does not appear to understand the conditions under which it is possible to **multiply** the base. For example, the student writes: $2^3 \times 3^3 = 6^6$ or $2^3 \times 3^3 = 6^9$ or takes 2^3 to be 6.	• What is the value of 2^3? What is the value of 3^3? What is the value of $2^3 \times 3^3$? What is the value of 6^6?
Student applies the property $(a^m)^n = a^{mn}$ incorrectly The student does not appear to understand the conditions under which it is possible to **multiply** *or* **divide** exponents. For example, the student writes: $4^3 = (2^2)^3 = 2^5$ (Q1e) *or* $(3^3)^3 = 3^6$ (Q1f)	• What does $(2^2)^4$ mean? Can you write it out as a product of integers? Is this equal to 2^6?
Student misinterprets a^{-n} or fails to use negative exponents. For example, the fails to answer (Q1i) or thinks that 7^{-1} is the smallest number (possibly -7) in (Q2).	• What is the value of 2^5? What is the value of 2^6? What is the value of $2^5 \div 2^6$? How can you write that as a power of 2? • Can you explain the difference in meaning of 7^{-1} and -7?
Student confuses a^0 and 0^a and believes they are both zero. For example, the student places 6^0 and 0^6 in the same answer box (Q2).	• How does each row in this sequence lead to the next row? What does the last row tell you? $2^4 \div 2^1 = 8 = 2^3$ $2^3 \div 2^1 = 4 = 2^2$ $2^2 \div 2^1 = 2 = 2^1$ $2^1 \div 2^1 = ? = 2^?$ • How would you write 0 x 0 x 0 x 0 using exponents? What is its value?
Student provides insufficient justification For example: The student correctly completes the missing exponent in Q1e giving only the justification that $4 \div 2 = 2$ so $3 \times 2 = 6$.	• Can you explain why the exponent has doubled? Does your method always work? Is 8^2 the same as 2^8? (i.e. $8 \div 2 = 4$ so $2 \times 4 = 8$?) • Can you express 4 as a power of 2? [2^2]

3.3 Answers

Many teachers like students to keep a record of their work from this lesson in their books. Figure 3 below shows a record sheet designed for use by students.



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AIMS

Applying properties of Exponents: Student record sheet

- For each group of cards, write all the expressions in the Expressions column and all the single exponents in the Single exponents column.
- You should have exactly 10 groups.
- The order of the groups does not matter.

Group	Expressions (E cards)	Single exponents (S cards)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Figure 4: Student recording sheet

The answers to the matching task can be found in the full lesson notes and are also provided on the following page in Figure 4.

Expressions		Single Exponents	
E12 $2^3 \times 2^3$	E10 $(2^3)^2$	S6 2^6	S10 4^3
E2 $3^2 - 2^3$		S5 2^0	
E7 $3^2 + 3^3$	E1 $2^2 \times 3^2$	S8 6^2	
E4 $2^2 \div 2^3$		S4 2^{-1}	
E6 $2^2 - 2^2$		S9 0^2	
E8 $4^2 \div 2^3$		S1 2^1	
E9 $2^3 \div 2^{-2}$		S2 2^5	
E3 $2^2 + 2^3$	E11 3×2^2		
E13 $5^2 - 3^3$		S3 $(-2)^1$	
E14 $(3^2 \times 2^2)^2$	E5 $6^8 \div 6^4$	S7 6^4	

Figure 5: Answers to the matching task