FaSMEd

Raising Achievement through Formative Assessment in Science and Mathematics Education

Properties of quadrilaterals

Subject: Mathematics

Age of students: 14 - 15 years

Used Technology: Card

Functionalities: Sending and displaying

Time: 1 or 2 lessons

FaSMEd partner: African Institute for Mathematical Sciences Schools Enrichment Centre

Short Abstract: This lesson is first about using properties of quadrilaterals to identify particular shapes and second about identifying the smallest number of properties needed to define a shape. After a short introduction, during which the teacher asks students to give the properties of a square, the students work in small groups. They are given strips of small cards; on each strip properties of a given quadrilateral are given. Students sketch the quadrilateral and later determine the smallest number of properties needed to define the quadrilateral. After the activity the teacher holds a class discussion aiming to go over the answers and elicit something about the students' understanding.

1. Content

This lesson is about properties of quadrilaterals and identifying the smallest number of these properties needed to define a shape.

2. Activity

2.1 Aims

This lesson intended to embed and reinforce knowledge about quadrilaterals.
In particular, students

- name and classify quadrilaterals according to their properties
- identify the minimal information required to define a quadrilateral
- sketch quadrilaterals with given conditions.

2.2 Structure / Methodology

Time needed: 60 to 90 minutes

Materials needed:

- Mini whiteboards for use in the introduction and during the lesson
- Strips of small cards for the students to use (one set per group)
- Prestik or blu-tak
- Scissors (one pair per group)
- A3 paper (one sheet per group)
- Instructions (one per group)
- Big cards for use in the wrap-up whole class discussion

2.3 Lesson plan

This is an abbreviated lesson plan, adapted from the one found at http://map.mathshell.org/lessons.php?unit=7325&collection=8.

Introduction

Hold a class discussion in which you revise what is meant by a quadrilateral and ask the students to give you the properties of a square. They may come up with a range of properties such as:

Four sided shape	Two pairs of parallel sides
Closed shape	Four right angles
All sides are equal	Two equal diagonals
Opposite sides are parallel	Diagonals bisect each other

Discuss which properties apply only to squares and which apply to one or more other quadrilaterals. (e.g. rectangles and squares have two equal diagonals).

Ask the students to suggest one pair of properties that define a square (e.g. four right angles and four equal sides).

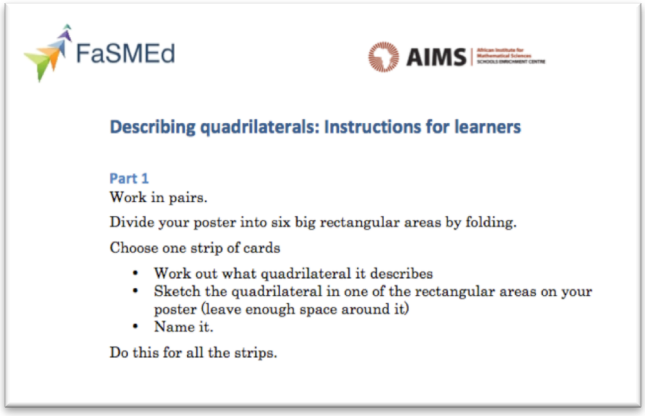
Remind students of the conventions we use to indicate parallel and equal lines, right angles and so on.

Pair work (Part 1)

Ask the students to work in pairs. To each pair, give a sheet of A3 paper, a set of strips of cards (see Figure 1), scissors and a Part 1 instruction sheet (see Figure 2).

A1 The diagonals of the shape are equal	A2 The shape has at least one side that is 5cm long	A3 The diagonals of the shape bisect each other at right angles	A4 The shape has 4 equal angles	A5 The shape has two pairs of parallel sides
B1 The shape has at least one side that is 4cm long	B2 The diagonals of the shape bisect each other	B3 The shape has 4 equal angles	B4 Opposite sides of the shape are equal	B5 The shape has at least one side that is 6cm long
C1 The diagonals of the shape are not equal	C2 The shape has at least one side that is 12cm long	C3 The shape has at least one side that is 7cm long	C4 The shape contains at least one 55° angle	C5 Opposite sides of the shape are parallel
D1 The diagonals of the shape bisect each other at right angles	D2 All four sides are equal	D3 The shape contains at least one 70° angle	D4 Opposite sides of the shape are parallel	D5 The shape has at least one side that is 7cm long
E1 The shape has at least one side that is 5cm long	E2 One diagonal bisects the other diagonal into two 2cm segments	E3 The shape has two pairs of equal sides	E4 The diagonals of the shape intersect each other at right angles	E5 The shape has at least one side that is 4cm long
F1 The shape contains exactly one pair of parallel sides	F2 The shape has more than one side that is 10cm long	F3 The shape contains at least one 60° angle	F4 The shape has a side that is 6cm long	F5 The shape contains a pair of opposite sides that are equal

Figure 1: Small strips of cards for students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Describing quadrilaterals: Instructions for learners

Part 1
Work in pairs.
Divide your poster into six big rectangular areas by folding.
Choose one strip of cards

- Work out what quadrilateral it describes
- Sketch the quadrilateral in one of the rectangular areas on your poster (leave enough space around it)
- Name it.

Do this for all the strips.

Figure 2: Instructions for students (Part 1)

Show the students how to fold their A3 sheet of paper into 6 equal parts.

For each strips of cards students should work out what quadrilateral it describes. On the A3 sheet of paper, they sketch the quadrilateral and name it. Make sure that they leave plenty of space below each sketch to stick cards on for part 2. Students can also be encourage to indicate the properties of each quadrilateral they draw.

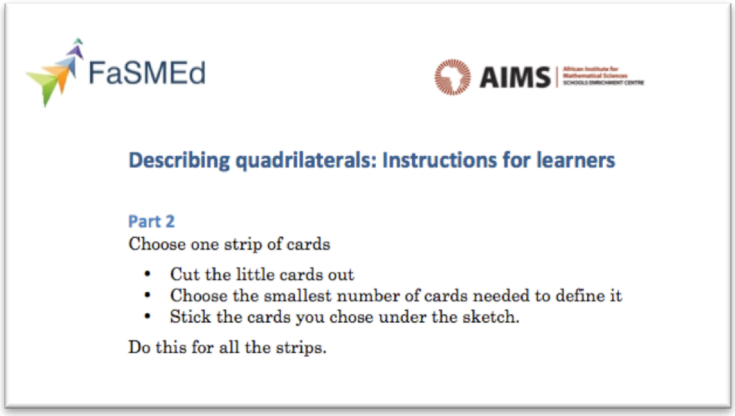
As you walk around, listen to the students' discussions and ask them questions if they appear to be stuck. Avoid telling them what to do (in terms of the maths) but ask them questions to lead them towards getting to the answer by themselves.

If time allows (Part 2)

Hand out Part 2 of the instructions (Figure 3) for students.
They should choose one strip of cards and

- cut the little cards out
- choose the smallest number of cards needed to define the quadrilateral
- stick the cards they chose under the sketch.

They should do this for all the strips.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Describing quadrilaterals: Instructions for learners

Part 2
Choose one strip of cards

- Cut the little cards out
- Choose the smallest number of cards needed to define it
- Stick the cards you chose under the sketch.

Do this for all the strips.

Figure 3: Instructions for students (Part 2)

Discussion

For each strip of cards, discuss the quadrilateral that was defined as well as its properties.

If students have completed part 2, ask for a volunteer to come to the board, sketch one of the quadrilaterals and stick the big versions of the little cards they chose to define the quadrilateral on the board. Repeat until all the sets of cards have been used.

Discuss the students' suggestions throughout. Emphasise that more than one set of cards can be chosen to uniquely define a quadrilateral.

2.4 Technology

- Mini whiteboards for use in the introduction and during the lesson.
- Sets of small cards for use by the small groups of students
- Sets of big cards for use in the whole class discussion

All these technologies provide opportunities for formative assessment in that they help to make the students' thinking visible.

2.5 Aspects of Formative Assessment

The teachers assess the students' levels of understanding in real-time and act accordingly by asking questions, giving hints and sometimes telling them the answer. They clarify learning intentions, engineer effective discussions, provide feedback and activate students as resources for one another and for themselves. This takes place at all phases of the lesson.

In the main activity, students work in pairs to sketch quadrilaterals. The intention is that students will take turns to explain their thinking to the others in their group. Each student should engage with their peer's explanation and take responsibility for the understanding of others in their group.

If the students follow these instructions, it is likely that they will be assessing their peers' levels of understanding and acting accordingly, sometimes explaining their own reasoning, sometimes telling the peers what to do and sometimes asking questions (i.e. peer formative assessment). In this case they would be activating their peers as instructional resources.

3. Further Information

3.1 Background

This lesson is based on the one designed by the Mathematics Assessment Project, which can be found at http://map.mathshell.org/lessons.php?unit=7325&collection=8.

The lesson design includes the use of a pre-lesson assessment task, which teachers can use to gather information about the current levels of students' understanding and their different problem solving approaches. The use of this task is highly recommended but in practice many teachers do not have time to do so. A post-lesson task is also provided.

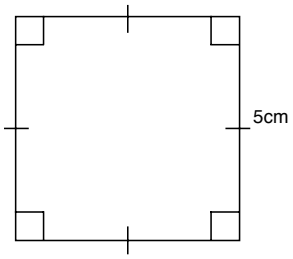
3.2 Common issues

The issues arising in the pre-lesson task are likely to arise in the main card activity and the table of common issues below provides some useful questions and prompts teachers could use to move the students' thinking on.

Common issues:	Suggested questions and prompts:
Understands different types of quadrilaterals as being distinct shapes rather some quadrilaterals being subsets of others For example: The student states that 'no' rectangles are squares	• What properties does a rectangle/square have? • Does a rectangle/square have all the properties of a square/rectangle? • Is it possible that one type of quadrilateral could be a special kind of a different quadrilateral? How could you tell from the properties if this was the case?
Assumes that the opposite sides of a rhombus are not parallel For example: The student states that 'no' rhombuses are parallelograms Or: The student states that 'some' kites are rhombuses Or: Fails to circle 'rhombus' as having at least one pair of parallel sides	• What do you know about the angles in a rhombus?
Assumes that a kite contains parallel sides For example: The student circles 'kite' as having at least one pair of parallel sides	• Does a kite have congruent sides? • Which sides in a kite are congruent?
Assumes diagonals that bisect must do so at 90° For example: The student circles just the square	• What does it mean for diagonals to bisect each other?
Assumes that the diagonals in an isosceles trapezoid bisect each other For example: The student provides an explanation that the diagonals of isosceles trapezoids bisect each other whereas non-isosceles trapezoids contain non-bisecting diagonals	• In what way is an isosceles trapezoid different to a non-isosceles trapezoid? • Draw in the diagonals of an isosceles trapezoid. What properties would the two triangles that are formed have if the diagonals were bisecting?
Provides little or no explanation For example: The student gives no reason for their choice of word and/or fails to explain their answers	• Which properties of (rectangles) do (trapezoids) not satisfy? • Can you convince me that a (rhombus) satisfies all the properties of a (parallelogram)? • What additional properties does a (square) have?

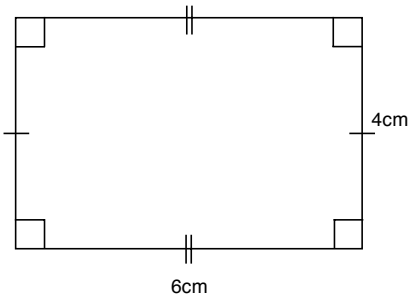
3.3 Answers

Shape A is a square:



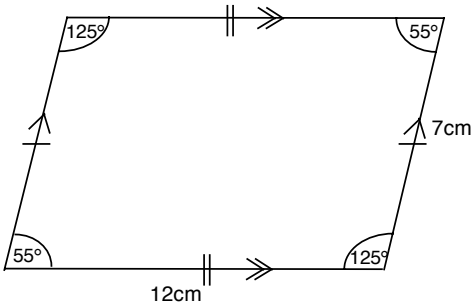
The minimal set of properties contains three cards, for example A2, A3 & A4 define the square.

Shape B is a rectangle:



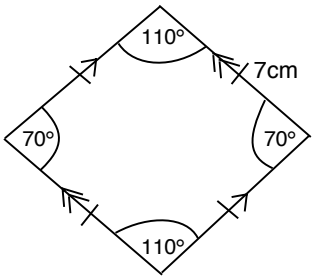
The minimal set of properties contains three cards, for example B1, B3 & B5 define the rectangle.

Shape C is a parallelogram:



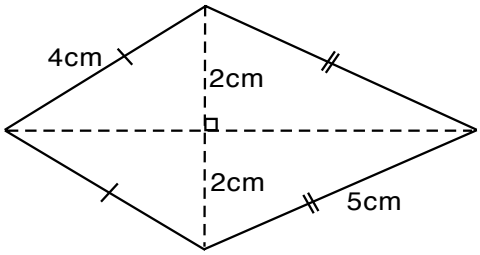
The minimal set of properties contains four cards, for example C2, C3, C4 & C5 define the parallelogram.

Shape D is a rhombus:



The minimal set of properties contains three cards, for example D2, D3 & D5 define the rhombus.

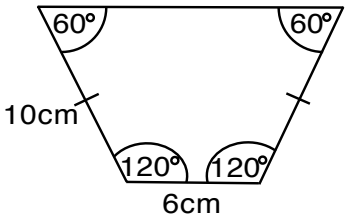
Shape E is a kite:



The minimal set of properties contains four cards, for example E1, E2, E4 & E5 define the kite.

No angles are given for **Shape E** so when students are sketching the kite they will not be able to label any angles on their sketch. However, it is possible to construct the kite from the information given.

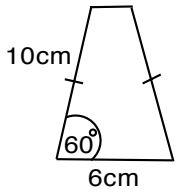
Shape F is an isosceles trapezoid:



The length of the longest side of the trapezoid is not given in the properties of **Shape F** so students will not be able to label the length of this side on their sketch. However, it is possible to construct the trapezoid from the information given.

All five cards are needed to define the trapezoid.

Note: Some students may sketch shape F as shown below:



This is not possible to draw.