FaSMEd

Raising Achievement through
Formative Assessment
in Science and Mathematics
Education

Teacher Guide
Digital Assessment Environment
Fractions

Marja van den Heuvel-Panhuizen, Mieke Abels, & Ilona Friso-van den Bos
Freudenthal Group, Faculty of Social and Behavioural Sciences
Freudenthal Institute, Faculty of Science
Utrecht University

Subject: Mathematics

Age of students: 10 - 14 years

Used Technology: Digital Mathematics Environment © FI – Peter Boon

1. Introduction

1.1 The FaSMEd project

FaSMEd is an acronym for ***F**ormative **A**ssessment in **S**cience and **M**athematics **E**ducation*. The FaSMEd study is a large, international research project in which universities from England, Ireland, Germany, Norway, France, Italy, South Africa, and the Netherlands participate. The study is financed by the European Union and aims at the development and research of formative assessment in mathematics and science education. The Dutch FaSMEd project is conducted by the Freudenthal Research Group belonging to the research program *Education and Learning* of the Faculty of Social and Behavioural Sciences at Utrecht University. In this Dutch part of the project, a digital assessment environment has been developed for mathematics education in grades 5 and 6 of primary school.

1.2 Formative assessment

Assessment is usually directly associated with the use of standardized instruments with which the achievement level of students in certain content areas can be measured, based on which one can make decisions about, for example, passing an exam or giving students access to a particular school type. This form of assessment is called summative assessment and aims to give a final judgement about the competence of a student.

Formative assessment concerns interim assessment. This form of assessment is focused on finding indications for further instruction. Formative assessment is in fact something teachers do continually during teaching. After all, proper teaching means that the provided instructions match the competence of the students, that the teacher knows which stumbling blocks there are, but also that the teacher knows what will help the students to (further) develop their understanding or skill.

Information about this can be collected in a number of different ways; for example, by asking questions, by observing students as they work alone or in groups, by assigning a series of teacher-developed problems, but also by administering an externally developed standardized test from a student monitoring system or a textbook, or having the students doing a test on the computer. All these methods of information collection are possible in formative assessment, as long as the assessment is focused on making didactical decisions. In other words, it is not in the first place the method of assessment that distinguishes between summative and formative assessment, but the intention with which it is conducted. An externally developed test can also be used formatively, but to yield true information for didactical decisions, it will have to produce more than a total score of correctly answered problems for each student. The Digital Assessment Environment that is being developed at Utrecht University within the FaSMEd project is not limited to providing such a total score, but also makes the students' strategies visible.

2. The Digital Assessment Environment (DAE)

2.1 DAE

The Digital Assessment Environment is a web-based environment with which teachers can collect information about their students' mathematics skills.

The DAE was built within the Digital Mathematics Environment (DME) (Boon, 2009). The DME is a software program developed by Peter Boon and his colleagues of the Freudenthal Institute at Utrecht University.

The recording facilities of the DME save the work of students and process it into an overview so that the teacher can have easily access to their students' work. The purpose is that teachers use the information about the mathematics skills that their students obtained to adapt their teaching to the students' needs. This may mean that teachers pay extra attention in class to a particular kind of problem, or that certain students receive additional instruction. The assistance provided can differ in nature, depending on the obstacles that students face. The special feature of the DAE is that it does not only make visible how many problems each student solved correctly, but also how the students solved the problems. To this aim the problems have been enriched with optional auxiliary tools (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2010; Van den Heuvel-Panhuizen, Kolovou, & Peltenburg, 2011) which the students can use to reach a solution. In this way, the DAE provides the teachers with important clues as to how students can be helped best to (further) develop an understanding or skill.

2.2 Assessment modules in the DAE

The DAE contains assessment modules by which the students' achievements in four mathematics domains can be assessed. The domains that have been chosen are those with which students generally have large difficulties, being: fractions, percents, and the metric system. The fourth mathematics domain is graphs. The decision to include graphs results from the wish of the FaSMEd partners to pay attention to graphs in each country.
For each of the four mathematics domains there are two tests: a Test A and a Test B. Each test consists of a series of six or seven problems. The problems in Test B are slightly more difficult than the problems in Test A, but the difference is small. Test A is intended for all students. Test B can be conducted, for example, to assess whether students still have difficulties after additional instruction.
The problems have been designed based on the reference levels 1F and 1S that have been set as standards for the end of primary education in the Netherlands (Noteboom, Van Os, & Spek, 2011) The six or seven problems in each test represent the core competencies that students have to achieve in the mathematics domains at hand.
To each problem various auxiliary tools have been added, like scrap paper, a bar, a ratio table, or a hint. These auxiliary tools are optional. The students may use them, but are not required to do so. However, by offering students this possibility, they get the chance to show what they are capable of with some support. In this way the 'zone of proximal development' is uncovered. To see which help is useful for the students is useful for the teacher to take further instruction steps.

3. Fractions

3.1 Didactical background information

Fractions also belong to the category of rational numbers. They express a part of a whole. A fraction can be the result of a fair distribution: sharing a cake with five people, so each gets $\frac{1}{5}$ of the cake ($\frac{1}{5}$ is an *common fraction* and is also called a *unit fraction* because the numerator is 1). But if you want to equally distribute six bars among four people, each gets $1\frac{1}{2}$ of a bar ($1\frac{1}{2}$ is called a *mixed faction*). So a fraction is not always a number between 0 and 1. It can also be a whole number and a part of something. This “of something” is very important for learning fractions. It means that fractions are used as named numbers. This gives students a lot of support in understanding fractions.

By means of sharing activities, students can get acquainted with *equivalent fractions* (fractions with the same value) in a natural and meaningful way. Sharing six bars in a fair way among four people can be done in different way. One can divide each of the six bars into four pieces. Each person then gets $\frac{6}{4}$ of a bar, which can also be expressed as $1\frac{1}{2}$ bar. Moreover, if each person would prefer smaller bits, each bar can also be divided into 8 pieces. Then everyone gets six times two pieces of $\frac{1}{8}$ bar, or $\frac{12}{8}$ bar. Understanding that $\frac{6}{4}$ bar and $\frac{12}{8}$ bar represent the same quantity of bar, means in fact having an understanding of a fraction as a rational number. A fraction can be expressed in various ways, but the ratio between the numerator and the denominator remains the same.

This rational aspect (the fixed relationship between numerator and denominator) makes the bar and ratio table especially suited models for understanding fractions and particularly for generating equivalent fractions.

Working with fractions on a number line, however, is also possible. Just like whole numbers, fractions can be places on a number line. A fraction is then treated as a number instead of as a ratio or as a part of a whole. By paying attention to fractions as numbers and placing them on a number line, the misconception can be avoided or overcome that fractions are also smaller than 1 (the “whole”). For example, students can be asked to place $12\frac{5}{6}$ on a number line and figure out in advance which number would be close to this fraction? Also, the number line is particularly appropriate for understanding and finding equivalent fractions. For example, the previous assignment could be followed by asking the students to find $12\frac{10}{12}$ on the number line too.

The latter issue, making equivalent fractions, forms the key to adding and subtracting fractions which have different denominators. All fractions cannot be added or subtracted just like that. For this, fractions should have the same denominator. This means that for each of the two fractions that have to be added or subtracted, an equivalent fraction must be found, so that the denominators of both fractions are the same. Once one has found the equivalent fractions, the addition or subtraction can be performed by adding or subtracting the numerators.

With respect to the multiplication and division of fractions, the primary school curriculum is usually limited to multiplying a (mixed) fraction by a whole number and dividing a (mixed) fraction by a whole number. This is in accordance with the Netherlands reference levels 1F and 1S. For these computations, a number line may provide assistance by visualizing multiplication and division as repeated addition and subtraction, respectively, and making the corresponding jumps on a number line.

3.2 Core competencies and test problems

Fractions		
Core competency	Test A	Test B
Comparing context-based fractions with different denominators	Problem 1 Click the largest part: $\frac{2}{3}$ part of a bar, or $\frac{5}{6}$ part of a bar How much is the difference? .... part of a bar	Problem 1 Click the largest part: $\frac{2}{3}$ part of a bar, or $\frac{2}{5}$ part of a bar How much is the difference? .... part of a bar
Adding context-based fractions with different denominators	Problem 2 $\frac{1}{8}$ of a bar and $\frac{3}{4}$ of a bar make together .... of a bar	Problem 2 $\frac{1}{6}$ of a bar and $\frac{3}{4}$ of a bar make together .... of a bar
Adding bare fractions and mixed fractions with different denominators	Problem 3 How much is $8\frac{1}{4}+\frac{2}{5}$?	Problem 3 How much is $8\frac{3}{4}+\frac{2}{5}$? ....
Subtracting bare fractions and mixed fractions with different denominators	Problem 4 How much is $6\frac{3}{4}-\frac{1}{3}$?	Problem 4 How much is $6\frac{1}{3}-\frac{3}{4}$? ....
Solving context-based division problems with fractions	Problem 5 How much is half of $1\frac{3}{4}$ bar? .... bar	Problem 5 Sharing $1\frac{1}{5}$ bar among three persons. How much does each get? .... bar
Solving bare multiplication problems with fractions	Problem 6 How much is $5 \times \frac{2}{3}$?	Problem 6 How much is $2\frac{1}{2} \times \frac{1}{2}$? ....

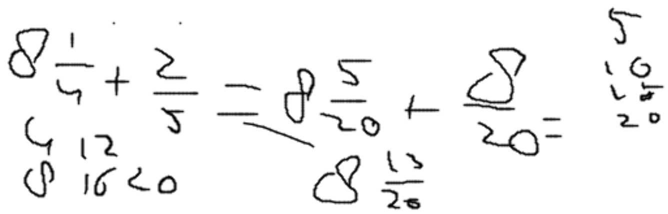
4. Auxiliary tools the DAE offers the students

Depending on the mathematics domain, students can use various auxiliary tools in the DAE for solving the problems: scrap paper (with or without a grid), a number line, a bar, a ratio table, or a hint.
Below follows a general description of these auxiliary tools, which can be used not only in the DAE but also in general, without a computer.

4.1 Scrap paper

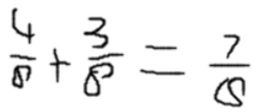
On scrap paper, students can make a drawing to clarify the problem to themselves. A very important function of scrap paper is also that it can support the calculation process. On scrap paper, students can write down the various calculation steps and the accompanying in-between answers. An example of this use of scrap paper can be found on the scrap paper of Student A. The 5, 10, 15, and 20 on the scrap paper show that the student searched for an appropriate denominator to make equivalent fractions (in fact, the student searched for the least common multiple).

Student A

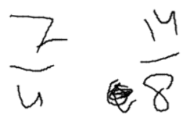


Further examples that show that scrap paper can reveal how (differently) students tackle a problem can be found on the scrap paper of Student A and Student B. The problem being tackled here is: 'How much is half of $\frac{3}{4}$ bar?' Both students gave the correct answer $\frac{7}{8}$.
Student B first took half of a whole bar and then half of an $\frac{3}{4}$ bar (or maybe the other way around), while student C converted $1\frac{3}{4}$ bar into $\frac{7}{4}$, then made $\frac{14}{8}$ out of it, to finally calculate mentally half of $\frac{14}{8}$.

Student B

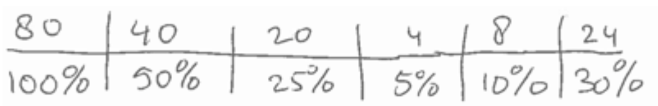


Student C

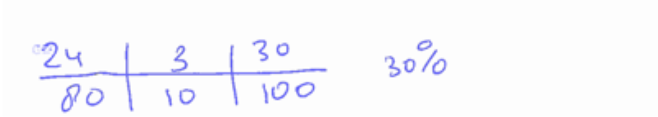


Scrap paper, of course, also offers students the possibility to draw a number line or a table as an auxiliary tool themselves.
Next the work of Student D and Student E is shown who drew a table when they had to calculate how many percent 24 out of 80 is.

Student D



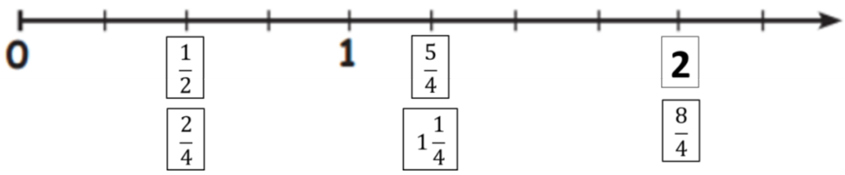
Student E



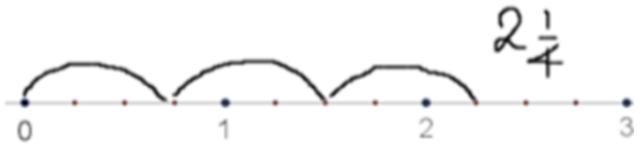
The nice thing of scrap paper is that it does not only help the students, but also the teacher. Scrap paper tells a teacher a lot about how a student has worked and often shows many things about the achievement level and understanding of a student. The work of Student D and Student E show how their understanding differs. Student D has chosen a rather long-winded, but correct approach, determining first that 80 is 100%, and then working towards 24 through miscellaneous intermediate steps with 30% as a result. Student E also has 30% as a result, but starts with the ratio of 24 to 80 and then works towards “so many out of 100”. Student E shows to have an understanding of the ratio aspect of percents, and to be aware of the need for a standardization to 100, while student D perhaps followed the rule “100% is all”.

4.2 Number line

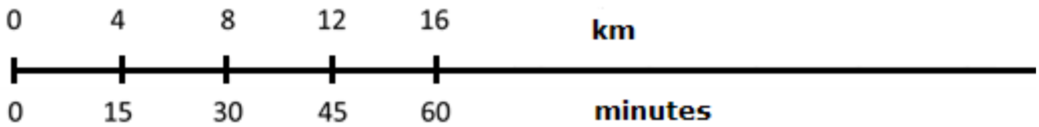
A number line is a simple but at the same time powerful model to represent numbers. This can be done on an empty number line that focuses on the order of numbers, or on a structured number line on which, in addition to the order of the numbers, the relative distance between them also matters. Such a structured number line can be used for all sorts of numbers: for whole numbers, for negative numbers, and for fractions. In the latter case, a number line is, as mentioned previously, a very appropriate tool for learning about equivalent fractions: $\frac{1}{2}$ is also $\frac{2}{4}$; $\frac{5}{4}$ is also $1\frac{1}{4}$; and $\frac{8}{4}$ is also 2.



Students cannot only use a number line to position and order numbers, but they can also use it to represent simple multiplications on. To solve 3 x $\frac{3}{4}$, students can make 3 jumps of $\frac{3}{4}$, thereby at the same time keeping track of where on the number line they have arrived and how many jumps they have already made.



Except for a single number line, there is also a double number line. In this case, the number line is used both on the top side and on the bottom side to notate numbers or values.



A double number line has similarities to a ratio table (see 4.4), but is also different from it. A double number line provides more visual support than a ratio table. An important difference between the two models is that on a double number line, the distances between numbers are visually represented. This makes that the double number line has some similarity with a measurement line, while the ratio table is more suitable to make calculations in convenient steps. A related difference is that on a double number line, the numbers are ordered from great to small, while this is not necessarily the case in a ratio table.

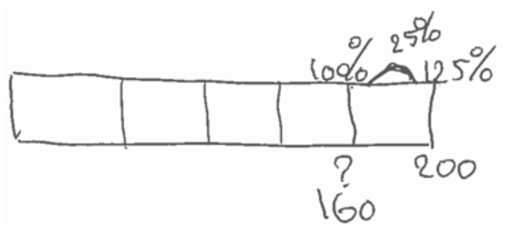
4.3 Bar

A bar actually consists of two connected number lines on which one can insert values on the top line and on the bottom line, which lines are proportionally related to one another. A bar is in fact a combination of two scale lines. For this reason, a bar is particularly appropriate to visually support work with rational numbers such as fractions and percents.
For a problem about a battery that works for 120 hours when fully charged and in which students have to find for how many hours it will work when charged for 40%, Student F used the bar as follows.

Student F

In comparison to the ratio table (see: 4.4), the bar is not a tool for calculating percents. It rather is a tool to grasp the problem situation. Because of the double-tracked representational character of the bar, one can both indicate on it for how many percent the battery is charged, and the number of hours the battery will still work. Starting from the 100%, Student F determined that at 50%, the battery will still work for 60 hours. The student then divided the 50% into 5 pieces of 10% each, which made 12 hours for each piece, and finally found that when the battery is charged 40%, it will still work for 48 hours.
Of course, not all numbers with which students have to work for percent problems are that suitable for doing all kinds of smart calculations. In problems with numbers that are less nice to handle, the bar can be used to estimate an answer, which can then provide a hold for doing a precise calculation.
In a problem about a change in percents, the bar can also be of help, not primarily for supporting the calculation process, but for supporting the thinking process. An example of such a kind of help is shown next. The problem is about a school that has 200 students this year, which is 25% more than the number of students last year. The question is how many students the school had last year.

Student G



As is shown on Student's G use of the bar, this student is quite aware that compared to the previous year the current number of students had increased 25%. So this means that the point of departure, the number of students last year, was 100%, and that the current situation is 125%. If 125% equals 200 students, then the bar can be divided into 5 equal parts to determine what 100% is. When doing a calculation this means that 200 has to be divided by 5 and then this answer has to be multiplied by 4.

4.4 Ratio table

The ratio table is primarily a tool for calculation; for example, in a problem such as “16 out of 20 newspaper photographs are in color, how many percent is that?” To calculate the answer of this problem the ratio table can be used in many different ways, as long as the ratio (between the part and the whole, or the percent and the number that belongs to this percent) remains the same. Below, there are some examples of student work.

Student H

part	16	8	80
whole	20	10	100

Student I

percent	100%	10%	80%
number	20	2	16

4.5 Hint

A hint can be used to help students in finding a solution. Such a hint can consist of suggesting the students how they can start to solve a problem. For example in a problem about a sports field which is 50 meters wide and 150 meters long, and the students have to find how many kilometers one has ran after ten rounds, the hint was “First calculate how many meters is one round.”

5. Test data provided to the teacher by the DAE

The DAE shows for every test for each problem which answer each student gave and whether this answer is correct or incorrect. Moreover, the DAE shows whether a student calculated the answer mentally or whether the student used an auxiliary tool, and if so: which auxiliary tool was employed. The DAE processes all these data into a class overview. This way, the teacher can see straightaway how the entire class performed on a test. In addition, the teacher can zoom in on the work of individual students. This means that the teacher can see, for example, what was written or drawn on the scrap paper, how the bar was used, or how a student applied the ratio table.

Which data are useful for making didactical decisions will not be the same for every teacher and every class. The teachers can use the test data that the DAE provides according to their own needs. The scheme below lists a number of questions for which teachers can get answers based on the test data. When using the test data three points of interest can be distinguished:

- the class as a whole
- individual students
- the learning opportunities offered by the textbook curriculum that is used.

A class overview can reveal, for example, that only very few students have mastered the competencies for a particular mathematics domain, while this domain should be feasible for the majority of the better performing students. Moreover, in the overview it can also become apparent that the bar has helped many students in the percent test, but that certain weaker performing students calculated their answers mentally. These are all findings that help a teacher to make didactical decisions, be it for the entire class or for individual students.

A particular point of interest concerns the mathematics textbook that is used in class. Based on class overview of the results of the students connected to the key problems and auxiliary tools included in the DAE, the teacher can investigate whether the textbook offers sufficient learning opportunities to the students. Does the mathematics textbook pay enough attention to all of the core competencies and the auxiliary tools?

Point of interest	Questions that can be answered using the test data from the DAE
My class as a whole	How is the **success rate** of my classroom in the domains of percents, fractions, metric system, and graphs? On which domains does my class score highest and on which domains lowest? Does my class **make use of the auxiliary tools**, or are problems generally solved mentally? How do my students use the auxiliary tools? Which proportion of problems is solved correctly by my class when the calculation is done mentally, and which proportion is solved correctly when students use an auxiliary tool? Which auxiliary tools are used most often in my class and which are used least often? Which auxiliary tools are used most often in my class in the domains percentages, fractions, metrics, and graphs? Which auxiliary tools are used most effectively in my class in the domains of percents, fractions, metric system, and graphs?
Individual students from my class	Which students perform below or above average in percents, fractions, metrics, and graphs, in terms of **accuracy**? Which students used a certain **auxiliary tool** more or less often than the rest of the class in certain domains?
My mathematics textbook	Do the **core** competencies for the domains percents, fractions, metrics, and graphs come up for discussion in my mathematics textbook? Which **auxiliary tools** come up for discussion in my mathematics textbook?

6. References

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