FaSMEd

Raising Achievement through
Formative Assessment
in Science and Mathematics
Education

Teacher Guide
Digital Assessment Environment
Metric System

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Subject: Mathematics

Age of students: 10 - 14 years

Used Technology: Digital Mathematics Environment © FI – Peter Boon

1. Introduction

1.1 The FaSMEd project

FaSMEd is an acronym for ***F**ormative **A**ssessment in **S**cience and **M**athematics **E**ducation*. The FaSMEd study is a large, international research project in which universities from England, Ireland, Germany, Norway, France, Italy, South Africa, and the Netherlands participate. The study is financed by the European Union and aims at the development and research of formative assessment in mathematics and science education. The Dutch FaSMEd project is conducted by the Freudenthal Research Group belonging to the research program *Education and Learning* of the Faculty of Social and Behavioural Sciences at Utrecht University. In this Dutch part of the project, a digital assessment environment has been developed for mathematics education in grades 5 and 6 of primary school.

1.2 Formative assessment

Assessment is usually directly associated with the use of standardized instruments with which the achievement level of students in certain content areas can be measured, based on which one can make decisions about, for example, passing an exam or giving students access to a particular school type. This form of assessment is called summative assessment and aims to give a final judgement about the competence of a student.
Formative assessment concerns interim assessment. This form of assessment is focused on finding indications for further instruction. Formative assessment is in fact something teachers do continually during teaching. After all, proper teaching means that the provided instructions match the competence of the students, that the teacher knows which stumbling blocks there are, but also that the teacher knows what will help the students to (further) develop their understanding or skill.
Information about this can be collected in a number of different ways; for example, by asking questions, by observing students as they work alone or in groups, by assigning a series of teacher-developed problems, but also by administering an externally developed standardized test from a student monitoring system or a textbook, or having the students doing a test on the computer. All these methods of information collection are possible in formative assessment, as long as the assessment is focused on making didactical decisions. In other words, it is not in the first place the method of assessment that distinguishes between summative and formative assessment, but the intention with which it is conducted. An externally developed test can also be used formatively, but to yield true information for didactical decisions, it will have to produce more than a total score of correctly answered problems for each student. The Digital Assessment Environment that is being developed at Utrecht University within the FaSMEd project is not limited to providing such a total score, but also makes the students' strategies visible.

2. The Digital Assessment Environment (DAE)

2.1 DAE

The Digital Assessment Environment is a web-based environment with which teachers can collect information about their students' mathematics skills.

The DAE was built within the Digital Mathematics Environment (DME) (Boon, 2009). The DME is a software program developed by Peter Boon and his colleagues of the Freudenthal Institute at Utrecht University.

The recording facilities of the DME save the work of students and process it into an overview so that the teacher can have easily access to their students' work. The purpose is that teachers use the information about the mathematics skills that their students obtained to adapt their teaching to the students' needs. This may mean that teachers pay extra attention in class to a particular kind of problem, or that certain students receive additional instruction. The assistance provided can differ in nature, depending on the obstacles that students face. The special feature of the DAE is that it does not only make visible how many problems each student solved correctly, but also how the students solved the problems. To this aim the problems have been enriched with optional auxiliary tools (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2010; Van den Heuvel-Panhuizen, Kolovou, & Peltenburg, 2011) which the students can use to reach a solution. In this way, the DAE provides the teachers with important clues as to how students can be helped best to (further) develop an understanding or skill.

2.2 Assessment modules in the DAE

The DAE contains assessment modules by which the students' achievements in four mathematics domains can be assessed. The domains that have been chosen are those with which students generally have large difficulties, being: fractions, percents, and the metric system. The fourth mathematics domain is graphs. The decision to include graphs results from the wish of the FaSMEd partners to pay attention to graphs in each country.
For each of the four mathematics domains there are two tests: a Test A and a Test B. Each test consists of a series of six or seven problems. The problems in Test B are slightly more difficult than the problems in Test A, but the difference is small. Test A is intended for all students. Test B can be conducted, for example, to assess whether students still have difficulties after additional instruction.
The problems have been designed based on the reference levels 1F and 1S that have been set as standards for the end of primary education in the Netherlands (Noteboom, Van Os, & Spek, 2011). The six or seven problems in each test represent the core competencies that students have to achieve in the mathematics domains at hand.
To each problem various auxiliary tools have been added, like scrap paper, a bar, a ratio table, or a hint. These auxiliary tools are optional. The students may use them, but are not required to do so. However, by offering students this possibility, they get the chance to show what they are capable of with some support. In this way the 'zone of proximal development' is uncovered. To see which help is useful for the students is useful for the teacher to take further instruction steps.

3. Metric system

3.1 Didactical background information

The metric system belongs to the domain of measurement. A characteristic of this domain is that it strongly reflects the relation between mathematics and the world around us. In measurement, mathematical means are being used to grasp our physical surroundings. Within the domain of measurement, students learn to describe and compare certain characteristics of objects and situations with the help measurement numbers. The measurable properties to which the students are introduced first are: length, area, volume, weight, time, and temperature. To quantify these properties, students can begin with natural measures (so many steps long) and later learn to work with standard measures (so many meters long). In the latter case, students also learn to use measuring tools. They also develop knowledge of measures (a kilometer is about three blocks) and use reference measures for making estimations (a door is about two meter, so the house will be that high). Length is a fundamental physical quantity with which the students gain experience first. This is later followed by understanding and calculating area and volume. One step further is learning to work with composed physical quantities such as speed (the relation between covered distance and needed time).

Dealing with the metric system differs from the act of measuring. Instead of measuring objects it is about how various units of measurement are related to each other. This is why for this sub-domain of measurement the term system is added.

This system of big and small units of measurement, with which measurable properties of objects can be expressed by a number, is built up very systematically, therefore one would expect that students have no difficulty mastering the metric system. However, the contrary is true. Many students in upper primary school, in which the metric system is taught, have considerable difficulties with the conversion of units of measurement. To understand this conversion, one needs to understand the underlying structure. Like in our number system, the factor 10 plays an important role in this. When you make the number 1 ten times as big, you get ten ones, or a ten. The metric system is constructed in a similar way. Each time you have ten of the same measurement unit, you can exchange it for one of a measurement unit that is ten times as big: 10 times 1 cm is 1 dm. Of course, this conversion is also possible in the reverse way: 1 dm is 10 times 1 cm. For understanding the metric system it is important that students know that when a measurement unit becomes 10 times as small, the measurement number becomes 10 times as big. Below this is displayed schematically.

x 10 x 10 x 10 x 10 x 10 x 10

Measurement number	1	10	100	1,000	10,000	100,000	1,000,000
Measurement unit	km	hm	dam	m	dm	cm	mm

÷10 ÷10 ÷10 ÷10 ÷10 ÷10

The opposite is the case when the measurement unit becomes 10 times as big. Then the measurement number becomes 10 times smaller each time, and one gets decimal numbers.

÷10 ÷10 ÷10

Measurement number	1	0.1	0.01	0.001
Measurement unit	m	dam	hm	km

x 10 x 10 x 10

The difficult thing about the metric system is that the ten structure only applies to all measurable properties that are linear, such as length (an exception is time, which is linear but does not have the ten structure). Area is two-dimensional. Area refers to a plane and not to a line. This means that in the case of area, a conversion from one measurement unit to the next implies an increase or decrease with a factor 100. With respect to area, one needs 100 pieces of 1 cm wide and 1 cm long to make pieces of 1 dm wide and 1 dm long. In short: 100 cm^2 = 1 dm^2. If one wants to make a piece of 1 m^2, one needs 10,000 cm^2.

x 100 x 100 x 100

Measurement number	1	100	10,000	1,000,000
Measurement unit	km^2	hm^2	dam^2	m^2

÷100 ÷100 ÷100

Volume is a measurable property with three dimensions, and is a spatial property. This means that the volume with a conversion of one measurement unit to the next comes with an increase or decrease of a factor 1000. In short: 1,000 cm^3 = 1 dm^3. If one wants to make a big cube of 1 m^3, one will need 1,000,000 cubes of 1 cm^3.

x 1000 x 1000 x 1000

Measurement number	1	1,000	1,000,000	1,000,000,000
Measurement unit	m^3	dm^3	cm^3	mm^3

÷1000 ÷1000 ÷1000

Except for this measure for the volume of objects (m^3 and the related larger and smaller measurement units), which are also known as cubic units of measurement, there is also a volume measure for liquids (and other substances without a fixed form such as garden soil). For this, we use liters (and the larger and smaller measurement units of this category). Although this is about the three dimensional physical quantity of volume, when making a conversion from liter to deciliter the factor of 10 applies. This may seem contradictory, but if one thinks of an amount of liquid in a very long and thin straw, one could imagine liter, dl, cl and ml to be linear measurement units, and understand the applicability of the factor 10.[1]

[1] Also remember that liter (and the smaller and larger measurement units in this category) is a measure of volume in itself, while “length” can be used for various different dimensions: a one-dimensional lenth (a line of 1m long), a two-dimensional “length” (a plane with an area of 1 m^2) and a three-dimensional length (a spatial shape with a volume of 1m^3). Also weight is only used in one way and “behaves in a linear way”; when making a conversion from one measurement unit to the next one, there is an increase or decrease with a factor 10. The fact that weight behaves linearly is well visible with, for example, a pull scale.

x 10 x 10 x 10

Measurement number	1	10	100	1,000
Measurement unit	liter	dl	cl	ml

÷10 ÷10 ÷10

Because both cubic units of measurement and the volume measures for liquids concern volume, conversions are possible between one measurement unit and the other. If one takes a container of 1 dm wide, 1 dm long en 1 dm high – so a container with a volume of 1 dm^3 – then exactly 1 liter of water can be poured in. Once one is aware of this, other conversions can be derived from this.

1 liter	1 dm^3	1000 cm^3	1.000.000 cm^3	1.000.000.000 mm^3
1 dl	0.001 dm^3	1 cm^3	1000 cm^3	1.000.000 mm^3
1 cl	0.000001 dm^3	0.001 cm^3	1 cm^3	1000 mm^3
1 ml	0.000000001 dm^3	0.000001 cm^3	0.001 cm^3	1 mm^3

Of course, students do not need to be familiar with all these conversions. According to the Netherlands reference levels 1F and 1F for mathematics at the end of primary school, it suffices if the students know: 1 dm^3 = 1 liter = 1000 ml (for reference level 1F) and 1 m^3 = 1000 liter (for reference level 1S).

For other conversions, it is also the case that students do not need to have them available. The focus should lie on learning about those units of measurement that are used mostly in daily life. The overview below contains the conversions that students at least need to have available.

1 km	1000 m
1 m	10 dm
1 dm	10 cm
1 m	100 cm
1 cm	10 mm

For students' understanding of the metric system and the structure of the measurement units it might be helpful to pay attention to the systematics of the prefixes.

		prefix	**meaning**
kilometer		kilo	thousand
hectometer		hecto	hundred
decameter		deca	ten
meter	**liter**		
decimeter	deciliter	deci	a tenth
centimeter	centiliter	centi	a hundredth
millimeter	milliliter	milli	a thousandth

3.2 Core competencies and test problems

Metric system		
Core competency	Test A	Test B
Converting measures of weight: g, kg	Problem 1 Click what is more: 5 kg or 7000 grams. How many grams is the difference? ... grams	Problem 1 A shopping bag weighs 3 kg. 40 grams are added. How many grams does the bag weigh now? ... grams
Converting measures of length: cm, dm, m, km	Problem 2 The height of a crate is 60 cm. A pile of five of these crates is … meters high.	Problem 2 A window is 2 m and 4 dm high. How many centimeters is the height of the window? ... cm
	Problem 3 A sports field is 50 meters wide and 150 meters long. How many kilometers ran the students after 10 laps? ... km	Problem 3 A sports field is 40 meters wide and 60 meters long. After how many laps did the students run exactly 4 km? After ... laps
Converting measures of area: m^2, cm^2	Problem 4 Click what is bigger: a terrace of 10 m^2 or a terrace of 800 dm^2. How many dm^2 is the difference? ... dm^2	Problem 4 How many m^2 of tiles are needed to make a terrace of 50 dm wide and 60 dm long? ... m^2
Converting measures of volume: dm^3, cm^3, liter, dl	Problem 5 Click which tank can hold the most: a tank of 30 liters or a tank of 28 dm^3. How many deciliters is the difference? … deciliters	Problem 5 Click which tank can hold the most: a tank of 4000 milliliters or a tank of 10 dm^3. How many deciliters is the difference? ... dl
Converting liter, dl, cl, and ml	Problem 6 One glass can 200 ml of lemonade. There are 3 liters of lemonade. How many glasses can you fill? … glasses	Problem 6 A bottle can hold 150 milliliters of perfume. A container contains 6 liters of perfume. How many bottles can be filled with this? ... bottles
Computations with complex magnitudes: speed	Problem 7 Tom lives 1000 meters from school. He walks about 4 kilometers in one hour. How much time can he needs to walk to school? ... minutes	Problem 7 After 1 hour and 20 minutes, Jasmine has cycled 20 kilometers. How many kilometers per hour is that? … km per hour

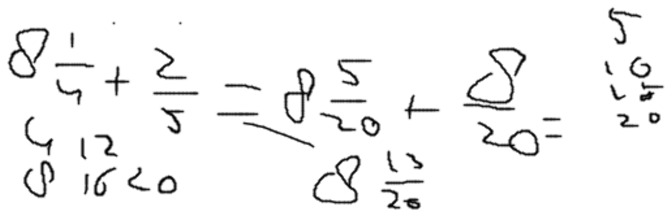
4. Auxiliary tools the DAE offers the students

Depending on the mathematics domain, students can use various auxiliary tools in the DAE for solving the problems: scrap paper (with or without a grid), a number line, a bar, a ratio table, or a hint.
Below follows a general description of these auxiliary tools, which can be used not only in the DAE but also in general, without a computer.

4.1 Scrap paper

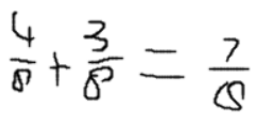
On scrap paper, students can make a drawing to clarify the problem to themselves. A very important function of scrap paper is also that it can support the calculation process. On scrap paper, students can write down the various calculation steps and the accompanying in-between answers. An example of this use of scrap paper can be found on the scrap paper of Student A. The 5, 10, 15, and 20 on the scrap paper show that the student searched for an appropriate denominator to make equivalent fractions (in fact, the student searched for the least common multiple).

Student A

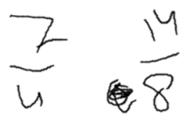


Further examples that show that scrap paper can reveal how (differently) students tackle a problem can be found on the scrap paper of Student A and Student B. The problem being tackled here is: 'How much is half of $\frac{3}{4}$ bar?' Both students gave the correct answer $\frac{7}{8}$.
Student B first took half of a whole bar and then half of an $\frac{3}{4}$ bar (or maybe the other way around), while student C converted $1\frac{3}{4}$ bar into $\frac{7}{4}$, then made $\frac{14}{8}$ out of it, to finally calculate mentally half of $\frac{14}{8}$.

Student B

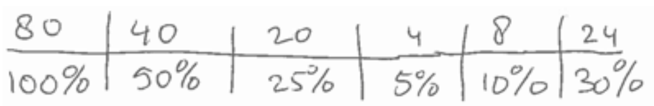


Student C

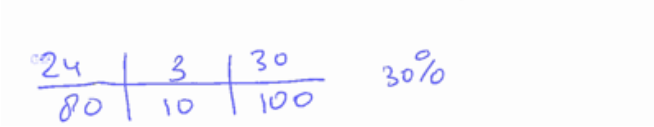


Scrap paper, of course, also offers students the possibility to draw a number line or a table as an auxiliary tool themselves.
Next the work of Student D and Student E is shown who drew a table when they had to calculate how many percent 24 out of 80 is.

Student D



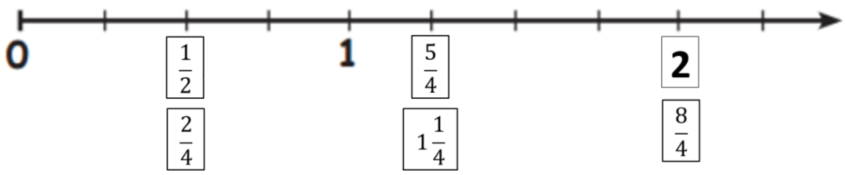
Student E



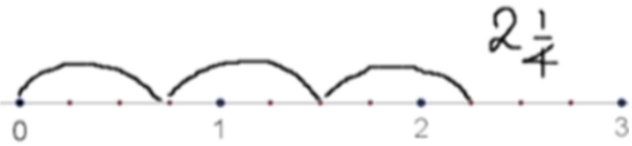
The nice thing of scrap paper is that it does not only help the students, but also the teacher. Scrap paper tells a teacher a lot about how a student has worked and often shows many things about the achievement level and understanding of a student. The work of Student D and Student E show how their understanding differs. Student D has chosen a rather long-winded, but correct approach, determining first that 80 is 100%, and then working towards 24 through miscellaneous intermediate steps with 30% as a result. Student E also has 30% as a result, but starts with the ratio of 24 to 80 and then works towards “so many out of 100”. Student E shows to have an understanding of the ratio aspect of percents, and to be aware of the need for a standardization to 100, while student D perhaps followed the rule “100% is all”.

4.2 Number line

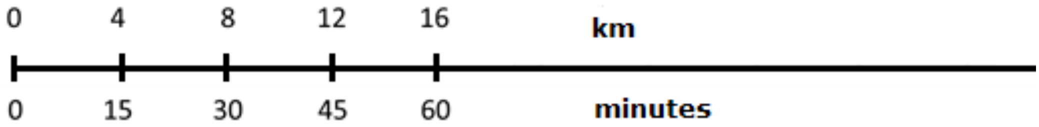
A number line is a simple but at the same time powerful model to represent numbers. This can be done on an empty number line that focuses on the order of numbers, or on a structured number line on which, in addition to the order of the numbers, the relative distance between them also matters. Such a structured number line can be used for all sorts of numbers: for whole numbers, for negative numbers, and for fractions. In the latter case, a number line is, as mentioned previously, a very appropriate tool for learning about equivalent fractions: $\frac{1}{2}$ is also $\frac{2}{4}$; $\frac{5}{4}$ is also $1\frac{1}{4}$; and $\frac{8}{4}$ is also 2.



Students cannot only use a number line to position and order numbers, but they can also use it to represent simple multiplications on. To solve 3 x $\frac{3}{4}$, students can make 3 jumps of $\frac{3}{4}$, thereby at the same time keeping track of where on the number line they have arrived and how many jumps they have already made.



Except for a single number line, there is also a double number line. In this case, the number line is used both on the top side and on the bottom side to notate numbers or values.



A double number line has similarities to a ratio table (see 4.4), but is also different from it. A double number line provides more visual support than a ratio table. An important difference between the two models is that on a double number line, the distances between numbers are visually represented. This makes that the double number line has some similarity with a measurement line, while the ratio table is more suitable to make calculations in convenient steps. A related difference is that on a double number line, the numbers are ordered from great to small, while this is not necessarily the case in a ratio table.

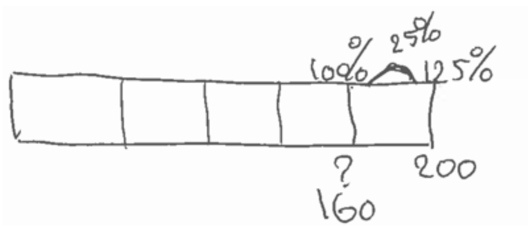
4.3 Bar

A bar actually consists of two connected number lines on which one can insert values on the top line and on the bottom line, which lines are proportionally related to one another. A bar is in fact a combination of two scale lines. For this reason, a bar is particularly appropriate to visually support work with rational numbers such as fractions and percents.
For a problem about a battery that works for 120 hours when fully charged and in which students have to find for how many hours it will work when charged for 40%, Student F used the bar as follows.

Student F

In comparison to the ratio table (see: 4.4), the bar is not a tool for calculating percents. It rather is a tool to grasp the problem situation. Because of the double-tracked representational character of the bar, one can both indicate on it for how many percent the battery is charged, and the number of hours the battery will still work. Starting from the 100%, Student F determined that at 50%, the battery will still work for 60 hours. The student then divided the 50% into 5 pieces of 10% each, which made 12 hours for each piece, and finally found that when the battery is charged 40%, it will still work for 48 hours.
Of course, not all numbers with which students have to work for percent problems are that suitable for doing all kinds of smart calculations. In problems with numbers that are less nice to handle, the bar can be used to estimate an answer, which can then provide a hold for doing a precise calculation.
In a problem about a change in percents, the bar can also be of help, not primarily for supporting the calculation process, but for supporting the thinking process. An example of such a kind of help is shown next. The problem is about a school that has 200 students this year, which is 25% more than the number of students last year. The question is how many students the school had last year.

Student G



As is shown on Student's G use of the bar, this student is quite aware that compared to the previous year the current number of students had increased 25%. So this means that the point of departure, the number of students last year, was 100%, and that the current situation is 125%. If 125% equals 200 students, then the bar can be divided into 5 equal parts to determine what 100% is. When doing a calculation this means that 200 has to be divided by 5 and then this answer has to be multiplied by 4.

4.4 Ratio table

The ratio table is primarily a tool for calculation; for example, in a problem such as “16 out of 20 newspaper photographs are in color, how many percent is that?” To calculate the answer of this problem the ratio table can be used in many different ways, as long as the ratio (between the part and the whole, or the percent and the number that belongs to this percent) remains the same. Below, there are some examples of student work.

Student H

part	16	8	80
whole	20	10	100

Student I

percent	100%	10%	80%
number	20	2	16

4.5 Hint

A hint can be used to help students in finding a solution. Such a hint can consist of suggesting the students how they can start to solve a problem. For example, in a problem about a sports field which is 50 meters wide and 150 meters long, and the students have to find how many kilometers one has ran after ten rounds, the hint was “First calculate how many meters is one round.”

5. Test data provided to the teacher by the DAE

The DAE shows for every test for each problem which answer each student gave and whether this answer is correct or incorrect. Moreover, the DAE shows whether a student calculated the answer mentally or whether the student used an auxiliary tool, and if so: which auxiliary tool was employed. The DAE processes all these data into a class overview. This way, the teacher can see straightaway how the entire class performed on a test. In addition, the teacher can zoom in on the work of individual students. This means that the teacher can see, for example, what was written or drawn on the scrap paper, how the bar was used, or how a student applied the ratio table.

Which data are useful for making didactical decisions will not be the same for every teacher and every class. The teachers can use the test data that the DAE provides according to their own needs. The scheme below lists a number of questions for which teachers can get answers based on the test data. When using the test data three points of interest can be distinguished:

- the class as a whole
- individual students
- the learning opportunities offered by the textbook curriculum that is used.

A class overview can reveal, for example, that only very few students have mastered the competencies for a particular mathematics domain, while this domain should be feasible for the majority of the better performing students. Moreover, in the overview it can also become apparent that the bar has helped many students in the percent test, but that certain weaker performing students calculated their answers mentally. These are all findings that help a teacher to make didactical decisions, be it for the entire class or for individual students.

A particular point of interest concerns the mathematics textbook that is used in class. Based on class overview of the results of the students connected to the key problems and auxiliary tools included in the DAE, the teacher can investigate whether the textbook offers sufficient learning opportunities to the students. Does the mathematics textbook pay enough attention to all of the core competencies and the auxiliary tools?

Point of interest	Questions that can be answered using the test data from the DAE
My class as a whole	How is the **success rate** of my classroom in the domains of percents, fractions, metric system, and graphs? On which domains does my class score highest and on which domains lowest? Does my class **make use of the auxiliary tools**, or are problems generally solved mentally? How do my students use the auxiliary tools? Which proportion of problems is solved correctly by my class when the calculation is done mentally, and which proportion is solved correctly when students use an auxiliary tool? Which auxiliary tools are used most often in my class and which are used least often? Which auxiliary tools are used most often in my class in the domains percentages, fractions, metrics, and graphs? Which auxiliary tools are used most effectively in my class in the domains of percents, fractions, metric system, and graphs?
Individual students from my class	Which students perform below or above average in percents, fractions, metrics, and graphs, in terms of **accuracy**? Which students used a certain **auxiliary tool** more or less often than the rest of the class in certain domains?
My mathematics textbook	Do the **core** competencies for the domains percents, fractions, metrics, and graphs come up for discussion in my mathematics textbook? Which **auxiliary tools** come up for discussion in my mathematics textbook?

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