



FaSMEd

Raising Achievement through
Formative Assessment
in Science and Mathematics
Education



Directed numbers

Subject:	Mathematics
Age of students:	13 - 14 years
Hardware:	iPads and IWB
Software:	Nearpod, App ' diagnosticquestions.com '
Functionalities:	Sending and Displaying
Time:	1 hour
FaSMEd partner:	University of Nottingham
Short Abstract:	The lesson focused on developing students' understanding of directed numbers including recognition of equivalent expressions, evaluation and word descriptions of applications in 'real life' contexts.



1. Content

In the lesson, students were expected to recognise equivalent expressions involving the addition and subtraction of directed numbers, to evaluate these expressions and to match word descriptions of problems involving directed numbers in 'real life' contexts to the mathematical expressions.

The lesson built on students' prior knowledge. These students had already been introduced to directed numbers and had some knowledge of how to add and subtract positive and negative integers. The lesson provided opportunities to address misconceptions and further develop understanding. Although the directed numbers remained within the same range (positive and negative integers) as their prior experience, the introduction of alternative forms of written expression and connections to 'real life' situations provided extensions to prior experience intended to enhance meaning.

2. Aims

The aims of the lesson were to:

- Develop students' conceptual understanding of the meaning of directed numbers;
- Extend students' understanding of different mathematical expressions involving the addition or subtraction of directed numbers;
- Develop students' understanding of, and fluency with, common processes used to add and subtract directed numbers;
- Enable students to simplify and evaluate numerical expressions involving the addition and subtraction of directed numbers;
- Enable students to construct and use appropriate mathematical models from 'real life' situations involving directed numbers.

2.1 Structure / Methodology

The lesson was based on the ideas within a task on directed numbers developed at the University of Nottingham as part of the Mathematics Assessment Project (MAP)¹. In the original lesson the conceptual difficulties associated with the addition and subtraction of directed numbers were discussed using a particular diagrammatic representation. This type of representation was incorporated into the Fasmed lesson plan for the purpose of explaining the concepts of addition and subtraction with negative numbers. Two of the three teachers used this representation in the lesson but the third chose to stay with the more familiar representation of the number line.

Prior to the lesson students were asked to complete a set of multiple-choice questions, devised by the teachers, using a computer or iPad at home and the web-based app: diagnosticquestions.com. This app marked the students' work but also required them to give a reason for their answer. A bank of correct answers was then made available to students electronically, including the reasons given by other students to explain these answers. Individual students could browse these and select a reason that they found helpful. In the

¹ <http://map.mathshell.org/download.php?fileid=1596>



screenshot below a student who initially gave a wrong answer has looked at reasons given by others and has selected one that they found useful.

Because two subtracting functions make a adding function. $-2 + 6 = 4$

When two minus signs are next to each other they cancel one and other out making a plus, therefore $-2+6=4$

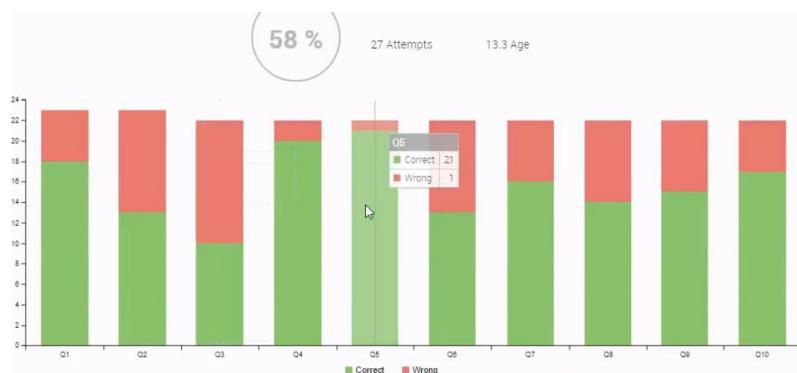
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✓ I LIKE THAT

MORE EXPLANATIONS

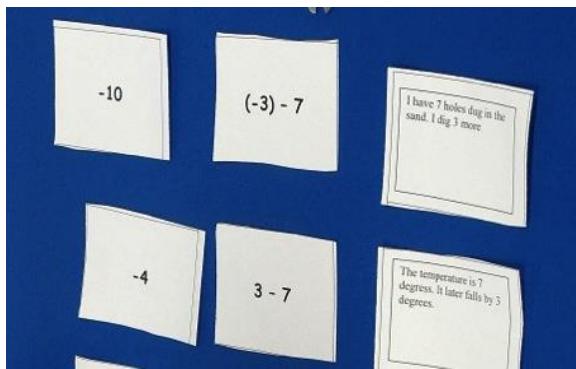
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The app also provided the teachers with access to extensive summary data about students' responses. One example of the type of summary available is shown below. Teachers could also drill down to view detailed reports about the performance of individuals.



On the basis of the pre-lesson work the teachers modified some of the questions planned for use in class.

Firstly students attempted a three way card-matching exercise, which was done using actual cards. This card-matching activity, devised by the teachers, was used to connect different expressions, solutions and 'real life' examples of adding and subtracting negative numbers. For example, students had to match "*The sea level starts at 7 metres below sea level. It then rises 3 metres*" with a card that says: $(-7) + (+3)$ and with a card that says: -4 . The activity was carried out in pairs or small groups and students were encouraged to work collaboratively. In following example, the students have matched the cards incorrectly.



Student work in progress was observed and photographed by some of the teachers, using an iPad, as they circulated around the class and questioned groups about their card matching. These photographs were later displayed on the interactive whiteboard (IWB) and used to initiate whole class discussion about common misconceptions and the validity of different card matches. The three teachers took slightly different practical approaches at this point with their classes but all made use of student work to prompt discussion. One of the teachers used a pre-prepared PowerPoint presentation of slides showing possible diagrammatic representations of the questions based on the MAP lesson model. The others utilised a combination of models and visual representations (such as the number line on the classroom wall) to provide explanations for students.

In the final part of the lesson, the students were asked to design their own cards for a card-sort activity and record these on a given template. The template consisted of a grid, which was either provided on a single sheet or supplied as sets of separate cards. Students had to enter numerical expressions, solutions and situations that matched into a row of three cards, thereby replicating the types of cards they had already been using during the card-sort activity. (An example of the template is given in the Appendix). Choices were constrained however, since the operations of addition and subtraction had already been entered, along with positive/negative signs for some numbers, into the numerical expressions section of the template. Students had, therefore, to think of a situation in which a particular type of calculation would take place and then evaluate the expression. During this activity the teacher again photographed students' work using an iPad.

The lesson concluded with the display of selected student work from the card design activity on the IWB and some whole class discussion in which students were questioned about the samples displayed. The teacher identified further misconceptions and provided explanations using the same models as they had chosen before.

2.2 Technology

The use of technology planned for this lesson included a 'send and share' function, plus some use of a 'process and analyse' function, in different formative assessment processes. These were utilised through the use of different software, with iPads or computers during the pre-lesson work and an iPad for the teacher within the lesson.

In the pre-lesson diagnostic work, the technology provided a range of summary reports but there was also the facility to 'drill down' to the individual student level for more detailed views of students' responses. This was useful information but meant that the teachers were faced with an extensive set of data that required a significant amount of mental processing and time in order to use the information effectively. They needed to process and select data from multiple reports and then make decisions on appropriate actions to take. Despite the



processing already provided by the app, the resulting array of information proved to be far more than the teachers could read, digest and use. The 'process and analyse' function provided by the technology was intended to assist the teacher in eliciting evidence of student understanding but the quantity of raw data made available to these teachers by the app presented them with a major task of further analysis and synthesis if they were to respond adaptively. From this example there is clear evidence that the amount and type of data provided by the technology needs linking to a clear purpose and a manageable process for teachers. Otherwise collecting the data becomes an unnecessary, or even counter-productive exercise with little value for formative assessment.

During the card sorting and card-design tasks, teachers chose not to use technology for the collaborative work but did utilise their own iPads to photograph and display student work on the IWB. The technology here performed a type of 'send and share' function but was primarily just a means of displaying information visually for the class to view, rather than featuring as an on-going mode of communication between teachers and individual students.

2.3 Aspects of Formative Assessment

The formative assessment strategies used in this lesson involved different approaches for the pre-lesson work and the lesson activities. Strategies had a clear focus on either the teacher obtaining information, for use in the planning and implementation of the lesson, or on students becoming instructional resources for each other.

Prior to the lesson the multiple-choice test (using the website diagnosticquestions.com) facilitated several formative assessment processes. When students used the site, the app generated detailed summaries and graphical representations for teachers, showing whole class and individual performance and students' explanations for their choice of answer. This data offered the teacher the opportunity to gain an overview of students' prior knowledge and common misconceptions to inform their lesson planning.

In this case, however, the app generated a large amount of secondary data for teachers. This increased the amount of information available, both about students' prior knowledge and also regarding their reasoning, but also made the process of using the data more complex. During the lesson, it became apparent that there were still many difficulties and misconceptions about operations with negative numbers, despite the diagnostic test, and the teachers found it difficult to address these satisfactorily. This highlights some important aspects of the formative process: firstly the need not just to extract information from students but also to obtain appropriate data in useable forms, and secondly the dependency on a teacher's skills in using the opportunities afforded by the data.

For students, the pre-lesson activity facilitated other formative assessment processes. As they submitted their answers these were marked electronically and if they chose an incorrect answer then the app supplied the correct answer. Students also had access to explanations provided by other users, so they could select one that they found helpful. The app therefore acted as an assessor of students' work but also provided guidance on how to re-think the problem. In this way, by providing direct feedback and guidance to students, without any teacher intervention, the activity enabled students to take greater ownership of their own learning, whilst indicating some of the criteria for success and suggesting how they might move forward in their thinking.

In the first section of the actual lesson, students worked collaboratively in groups to



complete the three-way card sort. This encouraged discussion between students that had a formative purpose. Students provided feedback to each other on their ideas, challenged each other's thinking and at times became instructors for their peers. These activities could potentially help clarify the criteria for success and move students' thinking forward in an environment where they had some ownership of their own learning.

The card-sorting task, however, proved challenging for these particular students and their lack of understanding of the mathematics impeded the formative assessment processes. Although the student groups in all three classes worked collaboratively and discussed the meaning of the expressions on the cards, progress was slow due to the multiple misconceptions that remained, despite the pre-lesson activity. Utilising students as instructional resources for each other was a well-intentioned formative assessment strategy but in this case, with this particular activity, the students' conceptual understanding did not appear adequate for them to become effective resources for others. The match of students' prior knowledge to the demands of the task needed more accurate assessment by the teachers to make the formative processes associated with this collaborative activity more productive.

In the second section of the lesson, formative assessment was linked to whole class discussion. The teachers photographed student work in progress using their iPads, displayed selected samples on the interactive whiteboard for the class to view and used these to generate discussion. Teachers questioned students about their own work, or that provided by their peers, thereby encouraging individuals to engage in self-reflection and act as peer assessors. At times, when students were able to offer coherent explanations of their own methods or solutions, students acted as useful instructors for others but the multiplicity of remaining misconceptions was still a problem and this, again, limited the effectiveness of the formative strategies.

The third section of the lesson involved a different task (the design of a card-sorting activity) but involved similar forms of student and teacher activity. Collaborative work in small groups, with support from the teacher, was followed by teacher-led whole class discussion and further explanations based on samples of student work. The formative assessment strategies and processes were also similar and the same difficulties arose, with multiple misconceptions still making progress difficult.

One of the other problems with this lesson was the mixed approach to developing conceptual understanding, by using a combination of different models to explain the addition and subtraction of negative numbers. In the planning of the lesson the teachers proposed to use the diagrammatic representation from the MAP project but in implementation some used the number line or other representations as well. This use of dual models led to some confusion and did not help move students' thinking forward.

Overall, the planned formative assessment strategies in this lesson were not as effective as anticipated by the teachers due to the mismatch between their expectations and students' actual understanding. The lesson activities did, however, perform a useful formative function in a different way because they provided ample evidence of students' understanding (or lack of it) that was valuable to inform their future lesson planning.

Within the formative assessment strategies described above, feedback was an intrinsic part of the processes involved. Firstly, feedback was provided by the web-based app to teachers and to students. For each party this was a vital part of a formative assessment process with a



clear potential impact on student learning. Teachers used the feedback provided by the program to adjust their lesson plans, whilst students engaged in a process of self-reflection and adjustments to thinking. The quality and quantity of the feedback provided by the app was, however, an issue in this lesson, in addition to the way in which it was used. Teachers were unable to use all the information provided and their adjusted lessons still proved too challenging for many students. Multiple misconceptions remained for individual students even after the feedback they had received from the app and the adjustments to the lesson made by teachers.

During the card-sorting and card-design tasks students gave feedback to each other on their ideas, explanations and suggested arrangements of the cards. This was supplemented by direct corrective feedback from their teacher at times or specific questions to prompt reflection on their decisions. Although the teachers worked hard at making timely and appropriate interventions, the quality of the feedback students received from their peers was a problem. Many students were confused and struggling to deal with misconceptions, which meant their feedback to others did not necessarily move thinking forward.

Further feedback was provided to some individuals from their peers and/or the teacher when their work was selected and displayed for class discussion. Here the teacher often intervened after feedback from individual students to correct thinking, clarify points raised or try to elicit further evidence of students' understanding. By adding their own explanations to the feedback provided by students, the teachers helped deal with misconceptions and move thinking forward. Their actions, however, shifted ownership away from the students and thereby affected the formative strategies that had been planned into the lesson.

3. Further Information

The transfer of activities from a paper-based approach to the iPad in this lesson is not viewed as a simple replacement of one resource for another but as a modification of a learning process. In the pre-lesson activity, the web-based app offered a quick way to obtain evidence of students' understanding that teachers found useful to pitch the lesson at the right level. For the card sorting, they considered that a paper-based approach was a better way of carrying out this type of collaborative activity but photographing student work provided a useful means of obtaining samples of student work for display and discussion. They did encounter some difficulties, however, in obtaining clear images.

The three-way card sort was particularly challenging for students, despite the diagnostic assessment prior to the lesson and subsequent adjustments to the lesson plan. Finding contexts to match numerical expressions is difficult for teachers as well as for students. Using mixed models for explanations of operations with directed numbers can confuse students, particularly if new models are used that are inconsistent with those already familiar to the class.

The students in this case identified some benefits in using the technology to display samples of their work on the IWB. In general, they believed this helped them to compare answers and methods so they could learn from their own mistakes and understand alternative methods.



4. References

Information on Nearpod available at <https://www.NearPod.com>.

MAP lesson available at <http://map.mathshell.org/download.php?fileid=1596>

Diagnostic questions app available from diagnosticquestions.com

Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in education*, 5(1), 7-74.

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