FaSMEd

Raising Achievement through
Formative Assessment
in Science and Mathematics
Education

The meatball problem

Subject:	Mathematics
Age of students:	14 - 15 years
Hardware:	iPads and IWB
Software:	Showbie, Google forms, Explain Everything™ (optional)
Functionalities:	Sending and Displaying
Time:	1 hour
FaSMEd partner:	University of Nottingham
Short Abstract:	The lesson involves a semi-structured approach to a problem about how many meatballs would fit into a pan of sauce before it overflows.

1. Content

During the lesson students worked on a semi-structured modelling task, based on a lesson attributed to Dan Meyer that appears on his website dy/dan.[1] Although the same basic format of the lesson was followed, several adaptations were made to incorporate different uses of technology that the teachers wanted to explore with respect to their effectiveness with certain aspects of this task.

Students were expected to have prior knowledge of the basic concepts of volume, capacity and surface area, in relation to common three-dimensional shapes. They were expected to be familiar with the most common units of measurement and understand relationships between these constructs and linear measurements such as radius, height and diameter. Although this was not the main aim of the lesson, there were opportunities that allowed them to explore these concepts and relationships further and clarify their meaning.

2. Activity

2.1 Aims

The aims of the lesson were to:

- Develop students' skills in posing and solving problems related to a 'real life' situation;
- Develop students' ability to identify relevant mathematical questions, variables and relationships in a given situation;
- Develop students' ability to apply mathematical knowledge, using appropriate mathematical models, expressions and terminology;
- Develop students' understanding of problem-solving strategies;
- Develop students' understanding of concepts related to the measurement of volume.

2.2 Structure / Methodology

In the lesson the students first viewed a short video clip from the website in which meatballs were being added to a pan of tomato sauce. The teacher then explained that the video showed something that used to be a textbook question and asked the students to suggest what that question might be. This was done in different ways by the teachers, either through an open class discussion with suggestions noted on the board, or by asking students to submit their questions using a Google form so that these could be displayed on the Interactive White Board (IWB).

The teacher then used the suggestions displayed to initiate a discussion. The purpose of this discussion was to involve students in making a decision about what particular question the class would adopt for investigation during the lesson. This discussion was guided by the teacher and did lead to the choice of a question that they had actually decided in advance, based on their expectations of the most common question suggested by students: *How many meatballs do you think will be needed in order for the sauce to overflow?*

The students were asked at this point to predict their answers to the problem and send these to their teacher using a Google form. These estimates/guesses were received by the

[1] http://blog.mrmeyer.com/2013/makeover-meatballs/

teacher but were only revealed to the class for comparison at the end of the lesson, when students had also spent time working out a more precise answer.

The teachers then prompted the students to start thinking about the problem but there were variations in the ways this was done. As suggested in the original Dan Meyer version, one teacher provided the students with specific information, from which they could work out an answer to the problem. This seemed to guide the students towards a particular method for solving the problem and therefore an alternative approach was used by the second and third teachers. This involved the them asking an open question such as “What information do we need in order to work out the answer?” The students then suggested items of information that they would require , some that were relevant (such as the volume of a meatball) and some that were not (such as the surface area of a meatball). There was then teacher-led discussion to refine terms and reach agreement about the information necessary to solve the problem. Asking the students to identify the information they needed removed some constraints, allowing them the freedom to investigate alternative approaches to the one implied from the given information. For example, they could decide to find the number of meatballs needed directly, by dividing the cylindrical volume of the space above the sauce in the pan, by the volume of a single meatball or, more intuitively, by iteratively adding the volume of the meatballs to the volume of the liquid and then seeing whether this exceeded the volume of the pan.

The students then worked collaboratively in pairs to work out an answer to the problem. They had the choice of working on a large sheet of paper or on their iPads. During the paired work the teacher went round the class, questioning students and answering their queries. This provided an opportunity to observe student work in progress and select samples that would be appropriate to display later for whole class discussion.

Towards the end of the session the students were asked either to open up a blank file, record their solution and send it to the teacher, or to take a photograph of their work and send it electronically to the teacher, or create a presentation using Explain Everything™ . The teacher then selected pairs of students to verbally present their work to the class, displaying their working on the IWB and explaining their methods. These samples of students work were then used these to generate class discussion regarding the methods used and their accuracy. In some of these lessons the teacher also had a sample of student work from a different class available to provide an example of a contrasting approach for the class to critique and discuss.

Finally the teacher revealed students’ initial estimates on the IWB and asked which assumptions were made at the beginning that had caused some estimates to be less accurate than others. Further class discussion followed to explore these assumptions and consider how they affected both estimates and solutions.

2.3 Technology

A variety of different methods were used in this lesson to make use of digital technology in a ‘send and share’ function, such as the use of iPads with Google Forms or *Showbie* that allowed students to send their responses (or work in progress) to the teacher. The data provided by the ‘send and share’ function was then used by the teacher in a number of ways and contributed to several different formative assessment processes.

For example, students’ responses to the initial request, to decide on the question that the video illustrated, were gathered, displayed and used in a class discussion (See Figure below).

The aim of this short activity was to agree on a problem that students would then work through during the lesson but the activity provided multiple opportunities for formative assessment, as detailed in the following section, that influence the learning trajectory for students during the lesson.

Figure 1: Students suggested questions

Meatballs 1 - RGT (Responses)

Timestamp	This used to be a textbook question, what was the question?
12/05/2015 08:58:44	How many meatballs does it take to make the pan overflow?
12/05/2015 08:59:48	How many meatballs will it take to make the sauce over flow?
12/05/2015 08:59:51	If you have 7 meatballs weighing 700g and then add them to 300g of tomato sauce, how much does the mixture weigh
12/05/2015 08:59:56	How many meatballs can you put in the sauce ?
12/05/2015 08:59:56	How many meatballs will it take to make the sauce overflow
12/05/2015 08:59:59	The ratio between meatballs and source
12/05/2015 09:00:08	How long did it take him to realise that his meatballs were out of date?
12/05/2015 09:00:08	How many meatballs can you fit in the sauce????
12/05/2015 09:00:52	What is the volume of the sauce after the meatballs have been added?
12/05/2015 09:00:55	What is the ratio of meatballs to sauce
12/05/2015 09:01:20	How mAny meatballs will it take for the sauce to overflow?

A Google form was used for students to record their estimates of the number of meatballs that would fit into the pan without it overflowing. This allowed each student to submit a suggestion and for the teacher to access this information quickly in tabular form for discussion later in the lesson.

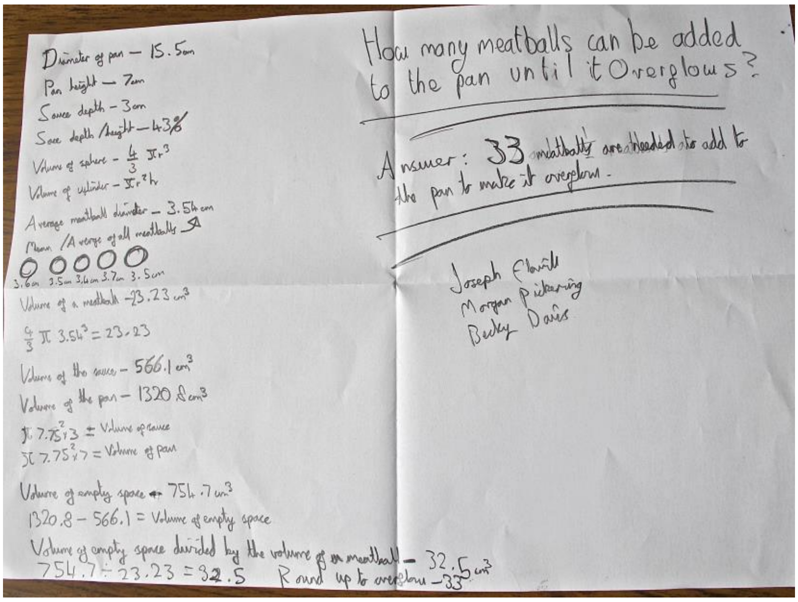
Figure 2: Students' estimates of how many meatballs it would take for the sauce to overflow.

Timestamp	Spike and Sanaa	How meatballs will take for the sauce to over flow?
13/05/2015 14:38:51	Sophia and zak	14
13/05/2015 14:38:51		100
13/05/2015 14:38:51	Will and Rhian	15
13/05/2015 14:38:54	sophie and Nathan	7
13/05/2015 14:38:57	Maisethan	11
13/05/2015 14:39:00	morgan+joe+becky	16
13/05/2015 14:39:02	Jack	8
13/05/2015 14:39:06	MC	8
13/05/2015 14:39:09		1
13/05/2015 14:39:30	MC	10
13/05/2015 14:39:36	jess	16
13/05/2015 14:39:46		14

The three teachers handled the need to determine the information required to solve the problem in different ways but there was again some use of a Google form to gather suggestions from students. This quickly facilitated the display of a collation of ideas in an accessible tabular form on the IWB. During the following discussion to identify the significant variables the teachers sometimes used visual images on the IWB to clarify terminology and understanding of the relevant concepts.

In preparation for the final class discussion the iPads were used to perform further 'send and share' functions. Students sent their work electronically from their iPads, either by first recording their solutions to the problem on an iPad or by them taking a photograph of their written work with their iPad. The teacher then selected and displayed samples of work from students so they could present their solutions.

Figure 3: A sample student solution



In each of these examples the technology performed a 'send and share' function that was then used as a basis for class discussion. In this way it was an essential part of the formative assessment process, providing feedback for the teacher on student thinking that they could use formatively but also facilitating the display of visual representations. The speed at which teachers could obtain the data and the accessibility of the forms in which it was supplied were features that had clear benefits for teachers.

2.4 Aspects of Formative Assessment

The formative assessment strategies planned into this lesson involved both teacher and student focused processes. Discussion, both for the whole class and between peers, played a significant part in these formative assessment processes and opportunities for peer assessment and self-reflection were evident through much of the lesson.

Opportunities to employ different aspects of formative assessment often occurred within one activity. For example, teachers used the initial discussion about the question to be posed to make some assessment of where students were in their understanding of the relevant mathematics. In addition, the discussion provided students with some indications of the key mathematical concepts that they would need to utilise in solving the problem. In this way the classroom discussion was primarily designed to develop students' understanding of the learning intentions and criteria for success but also helped elicit evidence of student understanding and provided some feedback move students' thinking forward.

The second whole class discussion focussed on what information was needed to solve the problem (i.e. establishing what needs to be done) but questions from the teacher also helped students gain a clearer view of where they should be going and the criteria for success. In addition, questions to students about the words they were using, their understanding of the concepts and the meaning of different mathematical terms also helped the teacher establish students' prior knowledge and conceptual understanding. For example, some students revealed a lack of understanding of the concepts of surface area and volume as they attempted to identify the information needed to solve the problem. The single activity of a class discussion therefore utilised a combination of different formative assessment strategies within a repeated teacher-led process of questioning, responding and providing feedback.

During these class discussions there were also opportunities for students to assess the validity of suggestions made by their peers and compare these to their own ideas. This peer assessment and self-reflection often resulted in adjustments to student thinking. Furthermore, the use of students' own suggestions regarding the question to be investigated and the information required was effective in increasing their sense of ownership of the problem, thereby functioning as a means of activation of students as owners of their own learning. Whole class discussions were therefore a significant source of opportunities for formative assessment in these lessons, for both teachers and students.

In between class discussions there were substantial opportunities in this lesson for student collaboration in pairs or small groups about the problem-solving task and this stimulated extensive discussion. Students discussed their strategies and often needed to justify their reasoning to their peers. There was evidence of students challenging others, adapting their own thinking and acting as instructors for each other during this collaborative work. Meanwhile the teacher observed students as they worked, informally assessed progress, answered questions and sometimes intervened with directed questions to help moving student thinking forward.

In the latter part of the lesson there was further discussion with the class following from students' presentations of their work. Allowing students to present their own work and respond to comments increased ownership of their own learning whilst other students had the opportunity to engage in peer assessment and self-reflection. These student presentations, and the use of other sample student work in the final class discussions, helped clarify the criteria for success for students with this type of problem-solving task and acted as a prompt to move thinking forward, in terms of the value of exploring alternative methods when attempting similar problems in the future.

The final comparison of initial estimates to actual solutions was a further opportunity for self-reflection, regarding the reasons for different answers and assumptions made. Although sometimes curtailed due to the lack of time, this was an example of a strategy to engage students in a critical appraisal of their own work that again helped to clarify the criteria that would lead to success with similar tasks requiring problem-solving strategies.

Teachers and students received feedback frequently during this lesson, mainly through whole class discussions or collaborative work in pairs or small groups. In the first part of the lesson, teachers received feedback on students' suggestions of questions to be posed, and/or information required to solve the problem and used this as a basis for class discussion. There were opportunities for the use of several different formative assessment strategies during these discussions, using the feedback gained prior to the discussion by the teacher and the verbal feedback from students during the discussions.

During the collaborative paired or small group work students provided feedback to each other on their ideas and work in progress. This challenged thinking and prompted self-reflection, thereby enabling students to act as instructors and move thinking forward as a result of their own interactions. Meanwhile the teacher provided additional feedback verbally on student work in progress to move thinking forward as they observed, questioned or responded to queries from students.

The later class discussions involved further feedback, from teachers and students, on student work. Students who presented their solutions received direct and specific feedback, from their peers and the teacher, regarding their methods and accuracy. Students also conveyed useful feedback to the teacher on their understanding for the problem through comments on the methods used by others. The final discussion, in which students compared their estimates to their worked solutions, involved some feedback on accuracy that led to discussion about the assumptions they had made when estimating.

Within the lesson feedback was an essential part of the formative assessment strategies used. The impact on learning was however dependent on how the feedback was used by the teacher and students within an extended process of gathering data, interpreting the information and making adjustments.

3. Further Information

Feedback from the teachers indicated that the use of iPads in this lesson had been useful to them in several ways. In particular, the speed and visibility provided by the technology, regarding short answers to questions and detailed solutions for the problem, were particularly helpful features. The benefits were strongly linked by teachers to the role the technology played in supporting class discussions. These were seen as vital activities within the planned formative assessment strategies for the lesson and the technology performed a useful function in facilitating these discussions. A suggested outline guide, with key questions for teachers to use, is provided on the following page.

Lesson objective

To use and apply appropriate mathematical skills to solve volume problems.

Implementation

1) Introduce lesson with tagline 'this used to be a textbook question, what was the question?'. Watch video on IWB.
2) Google Form to collect responses. Display on board and discuss the responses with the class. Agree on a principal question 'How many meatballs will it take to overflow?' (various numbers anticipated)
3) Google Form, 'estimate how many meatballs you think will be needed in order for the sauce to overflow'. Keep results private (to reveal later), but ask for students names in form.
4) 'Since that is the question, what information would we need in order to work out the answer?' Give 5 minutes for discussion on this question, ask them to make a list for discussion and ask them to make a plan for answering the question.

Possible responses:

- Size of the pot - 'What do you mean by size?'
- The volume of sauce - 'Do we need to know the volume of sauce in order to work out how many more meatballs would fit in?'
- Height of the saucepan - 'Do we need to know the height of the saucepan in order to work out how many more meatballs would fit in?'
- Volume of pan - 'What information would we need in order to work out the volume of the pan'
- Size of meatball - 'What do you mean by size?'
- The volume of the meatball - ''What information would we need in order to work out the volume of the meatball'

After each piece of information is agreed upon, pass on the required information. Students should make notes on the A3 sheet of paper provided. Leave information on IWB.

5) Students to work out the answer to the problem on A4 sheet.
6) Once answers are complete, students present solutions. This can be done in various ways. For example, they might create a presentation in *Explain Everything*™ to show their solution.

Possible extension:

Ask the question: if all of the required meatballs were rolled up into one giant meatball, would it have fit in the pan?

A few groups will be selected to display their solutions and explain each point - class to critique. Selected by observation.

7) Display the correct answer. 'Did anyone get it right?' 'Why does the Maths not work?' Discuss the assumptions that were made that caused the answer to be wrong.
8) Reveal original guesses for discussion.

Conclusion

Example work displayed. Discussion of :

- Why inaccuracies arise and need to be identified.
- How to use mathematics when solving real life problems.

4. References

Original meatball problem available from http://www.101qs.com/2352-meatballs

Explain Everything™, information available at http://explaineverything.com/

Google forms, information available from https://www.google.com/forms/about/

Showbie available at https://www.showbie.com./

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