FaSMEd

Raising Achievement through
Formative Assessment
in Science and Mathematics
Education

Tessellations

Subject: Mathematics
Age of students: 10 - 11 years
Hardware: iPads and IWB
Software: 'Tessellation Creator' app
Functionalities: Sending and Displaying
Time: 1 hour
FaSMEd partner: University of Nottingham
Short Abstract: The lesson focused on clarifying concepts and terminology about 2-D shapes before students explored the concept of tessellation by carrying out their own investigations into which regular shapes tessellate.

1. Content

The main purpose of this lesson was to develop students' understanding of the meaning of tessellation through exploration and discussion. These students were already familiar with common regular and irregular shapes (e.g. triangle, square, rectangle, quadrilateral, pentagon, hexagon, octagon) but had not encountered the concept of tessellation before. The lesson started with verbal questions and some class discussion about prior knowledge to clarify concepts and terminology about 2-D shapes. Students then explored tessellations of regular shapes to develop conceptual understanding, using the 'Tessellation Creator' web-based app.

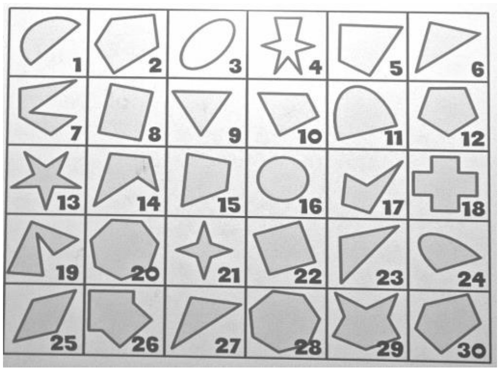
1.1 Aims

The lesson aims were to:

- Build on student's prior knowledge to widen their understanding of regular shapes and their properties;
- Develop students' understanding of the concept of tessellation;
- Enable students to identify shapes, or combinations of two shapes, that tessellate;
- Enable students to create their own tessellations using single regular shapes, or two regular shapes in combination;
- Develop students' skills in critiquing solutions to mathematical problems.

1.2 Structure / Methodology

The lesson commenced with the display of 30 two-dimensional shapes on the interactive whiteboard (IWB) and the teacher asked a series of questions to obtain information about students' prior knowledge and understanding. For example, students were first asked to write down the numbers of any pentagons from the screen shown below.

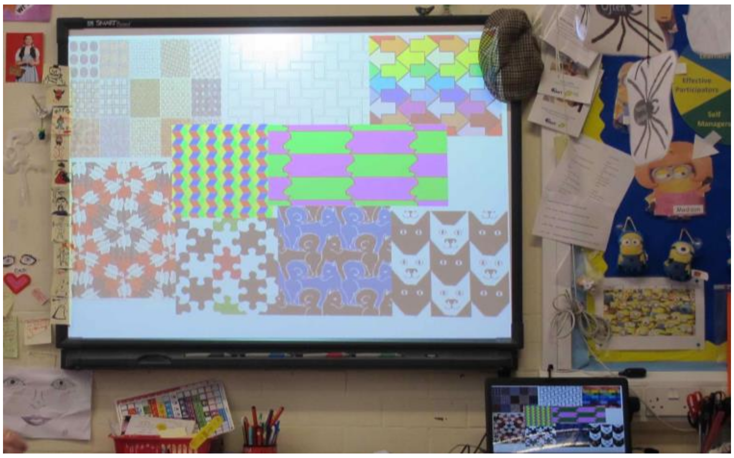


The students worked in pairs with a 'learning partner' (a formal arrangement into pairs used for most lessons) to answer the question and were then encouraged to compare their answers to others on the same table. This was followed by a whole class discussion in which the teacher asked individual students for their answers and added further comments to explain why certain answers were correct.

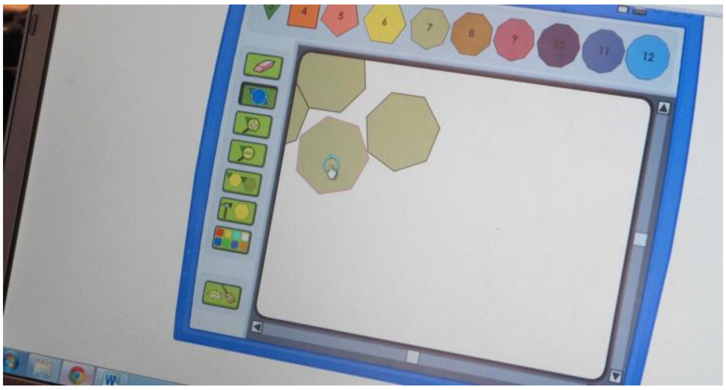
The teacher then asked the pairs to decide which of these shapes were regular pentagons? Again, after some paired discussion, there was a whole class discussion in which students made suggestions and the teacher questioned them further to extract explanations and clarify the reasons for their decisions. Further questions were then asked by the teacher, considered in pairs and discussed with the whole class: Which shapes are quadrilaterals?

Which of these quadrilaterals have right angles in them? Sometimes the students were asked to come to the board and point out particular shapes. The teacher often followed up students' suggestions with further questioning to clarify meaning.

At this point the class were asked to view some examples of tessellations on the IWB. The teacher asked what they observed about the images on the IWB. The students responded with suggestions such as "They are all the same shape", "Some are rotated", "They are different colours" and "They all 'net' together". The teacher questioned them further to draw out the idea of either the same shape or different shapes fitting together with no gaps.



The students were then introduced to the main task, which was to share laptops and use a web-based app 'Tessellation Creator' to find out what shapes tessellated. The app provided students with a range of regular shapes, from 3 sided to 12 sided (as shown in the following image) which they could replicate, move around, rotate and re-size.



Reference was made to how the task might be like trying to tile the hall floor in the school. First, the students were asked to make predictions, in pairs, of common regular shapes that would tessellate and write these on a 'post-it'. Most students soon decided that squares and triangles would tessellate but were less confident about pentagons, hexagons and octagons. Each pair then worked together on a shared laptop to test their predictions and see which shapes would actually tessellate. The students asked various questions during this activity to clarify, for example, whether the shapes could overlap and whether they needed to fill the gaps at the edge of the screen for a valid tessellation. After students had explored tessellations using just one shape at a time there was some discussion with the class about using a combination of two shapes and further time to explore this in pairs.

In the final part of the lesson the students were asked to show their most interesting tessellation on their laptop screen. The class walked round to view the work done by others and recorded their comments. Each student was then asked to write down something they had learned about tessellations on a 'post-it' and there was some whole class discussion in which students were asked to explain their 'theories' about tessellation. For example, some suggested that shapes with an even number of sides tessellated but those with an odd number did not. These suggestions were not explored further within the lesson due to time constraints but the students were asked to leave their predictions and statements of what they had learned for the teacher to collect and use in future planning.

1.3 Technology

In this lesson laptop computers were used with a web-based app to explore the concept of tessellation and find solutions to the question of which shapes would tessellate. The function of the technology was therefore not directly linked into a formative assessment process and did not perform a 'send and display' or 'process and analyse' function. Instead the app provided a learning space in which students could explore tessellations, although this was not a fully interactive environment since the technology provided only limited visual 'feedback' to students about whether the shapes were actually tessellating.

The visual representations did, however, cause students to reflect on their decisions, make adjustments and sometimes prompt new questions, such as whether the gaps at the edge of the screen needed to be filled, or if overlapping was valid. Some of these questions arose because the technology allowed certain actions without providing any feedback to indicate whether the result was acceptable as a tessellation. For example, the app allowed shapes to overlap without any hint that this was unacceptable as a tessellation. Interestingly, this was a question that did not normally arise with paper-based methods or the use of plastic/card shapes but seemed to follow from the way in which the app functioned.

The way the technology was used in the lesson did, however, encourage collaboration. Sharing laptops and working together in pairs meant that students discussed their work, challenged each other's ideas and made adjustments to their own thinking. The sharing of laptops therefore helped create a collaborative working environment in which formative assessment took place.

1.4 Aspects of Formative Assessment

In this case there were two dominant features of the lesson through which teachers implemented key formative assessment strategies. Firstly, the teachers used questioning extensively in whole class discussions to elicit evidence of students' understanding and clarify meaning. In addition, they created an environment in which students could work collaboratively to explore meaning together and exchange ideas.

In the whole class discussions the students were asked about their responses to questions that they had usually just discussed in pairs or small groups. This meant that they came to the class discussions with ideas that had already been subject to some feedback from their peers. Through this process some misconceptions and errors had already been discussed and thinking had been adjusted so students were well prepared for the class discussions. After students had given their initial responses, the follow-up questions used by teachers often probed more deeply for evidence of understanding and these students were able to respond with good explanations. In this way the teachers used the first section of the lesson diagnostically to determine where students were with respect to their prior knowledge and understanding. The students themselves were an important part of these discussions and their interactions with the teacher facilitated an effective formative process. This was a forum in which students received further formative feedback, whilst teachers gained useful information about students' thinking and exposed misconceptions.

The collaborative work in pairs and small groups was designed to foster discussions between peers that could also have a useful formative function. As students worked together and discussed their ideas they challenged each other's thinking and engaged in peer assessment. There was some evidence that this process resulted in some valuable self-reflection and adjustments to thinking for some students.

A strong theme within the lesson was the way in which ownership of learning was often shared with the students and this was done in three ways. Firstly, the teachers' approach to whole class discussion was to ask students to explain their responses, using students to clarify the criteria for success and act as instructional resources for others. Secondly, much of the lesson involved paired or small group work in which students could discuss and develop their own thinking as they became peer assessors and instructors for each other. Finally, asking students to conduct their own explorations into which shapes would tessellate meant that student were working in a semi-structured environment where they themselves were making decisions on how they tested their own predictions.

Feedback was an intrinsic part of the formative assessment processes described above and this included feedback from both teachers to students and students to peers. Much of this lesson had a strong student focus and students gave feedback to each other during the exploratory work in pairs and the whole class discussions. The distinctive feature of the students' feedback in this case was the quality of their explanations when questioned, particularly considering their age and mathematical experience. Their feedback contributed effectively to the planned formative assessment processes, informing the teacher about their understanding and challenging their peers to re-think ideas.

Feedback from the teacher was given to students during the whole class discussions to affirm or correct their thinking and also during the paired work. Through the way these teachers observed, listened and responded to students some effective processes were established that contributed to the quality of formative assessment.

2. Further Information

Although the lesson was planned for students aged 10-11 years, it was also adapted for use with younger groups of students aged 9-10 years and 8-9 years. In these classes one or more of the following changes were made to the first part of the lesson:

1. The students were asked the same questions but were provided with mini-whiteboards to record the numbers of the shapes they chose. They displayed these to show their answers and the teacher directed questions to individuals using their answers from the whiteboards.
2. The teacher commenced by using the IWB to reveal part of a shape and then asked the students to decide what the shape was. After some discussion in pairs a little more of the shape was shown and once students correctly identified the shape (e.g. a square) they were asked how they knew. The process was repeated with a different shape (e.g. a pentagon).
3. A limited range of just 6 shapes was shown on the IWB rather than 30. The students were asked to find different shapes, such as a hexagon and then asked to predict which of these shapes would tessellate.

The teachers chose to use the laptops in preference to iPads for this lesson since they anticipated difficulties for students when using a 'touch screen' for this particular activity. They commented on how well the students recalled previous work but were surprised at their predictions about which shapes would tessellate. This suggests that some students found it difficult to visualise shapes and manipulate these mentally. Using the app though seemed to make the task of testing out which shapes would tessellate fairly easy. The technology made the task much quicker compared to drawing tessellations by hand. The other advantage over plastic/card shapes was that they could easily be resized. The app therefore seemed to save time and also stimulated deeper thinking. For example, one teacher commented on how students' final statements on 'post-it' notes indicated that most students had understood the key principles of tessellation and many were beginning to conjecture why certain shapes would or would not tessellate.

The teachers found the technology was quite simple in its functions but also a little difficult for some students due to technical problems in the design. For example the rotations were not fine-tuned and students sometimes questioned whether they had a true tessellation or not. In contrast, conceptual issues that arose, like whether overlapping was allowed, became a useful teaching point that may not have become apparent if children had used plastic or cardboard shapes for this activity.

The following lesson plan provides further information about the basic structure and content of the lesson before adaptations were made.

Subject(s)	Maths	**Technical/Key Vocabulary**	Tessellation, 2D, properties, angles, vertices, edges, faces
Context	Revision week on transforming 2D shapes	**Resource** (inc TA deployment)	Laptops – 1 between 2 TA deployed to support specific pairs
Learning Objective	To understand what tessellation is and which shapes tessellate.		
Success Criteria (How will I know whether I've achieved the learning intention?)	• Do I know that tessellating patterns have no gaps? • Do I know the shapes that make regular patterns? – triangle, quadrilateral, regular hexagon • Have I experimented with semi-regular tessellations using more than 1 shape? • Have I produced a tessellating pattern to fulfil the brief? • Can I start to describe which shapes tessellate and why?		
Introduction	**Approximate Timing** 10 min	Recap names and properties of 2D shapes in pairs/groups – starter activity	
Main Activities For different groups LA/MA/HA	40 mins	10 mins - Main teaching – introduce idea of tessellation, demonstrating how shapes can be fitted together with no gaps. Show some examples of simple tiled patterns. Set context – hall floor needs retiling and should look attractive. Show website: http://illuminations.nctm.org/Activity.aspx?id=3533 5 mins - Ask children to predict in pairs which shapes will tessellate and cover the floor area. Children to jot down predictions on a post-it note. 5 mins - Let pairs experiment with using one shape at a time to discover which shapes tessellate. Pairs to feed back to table groups what they discovered – e.g. triangles, quadrilaterals, hexagons and discuss everyday examples of these tessellations. (Mixed ability pairings) 15 mins - Now ask children to investigate with a variety of shapes and how they could cover the floor area on their screen. Which combinations of shapes tessellate? Is there a way to tell if shapes will tessellate by looking at their properties.	

		Open-ended investigation to explore shapes and combinations. Group discussions about findings/difficulties/discoveries 5 mins -Pairs tour the room to look at other tessellating patterns and to leave feedback for a specific tessellation
Plenary (including key questions)	10 mins	10 mins - Feed back to whole class Look at a few examples on screen and take suggestions about what properties shapes need to be able to tessellate Refer back to work on interior angles of shapes
Support for students with **SEND:**	Mixed ability pairings TA/teacher support for specific pairs with SEND children	

3. References

Tessellation Creator available at https://illuminations.nctm.org/Activity.aspx?id=3533

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Wiliam, D. (1999). Formative assessment in mathematics Part 2: feedback. *Equals: Mathematics and Special Educational Needs*, *5*(3), 8-11.