USING STUDENTS' MISTAKES TO PROMOTE LEARNING

HANDOUTS FOR TEACHERS

Contents

Acknowledgement:

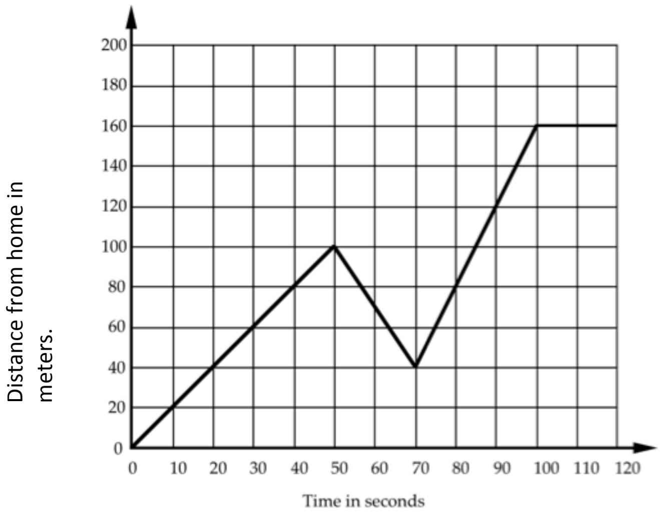
In preparing this material, we acknowledge the permissions given by the Bowland Charitable Trust to adapt the professional development resources, Bowland Maths (www.bowlandmaths.org), that they had previously funded the University of Nottingham to produce for the UK. This includes many of the handouts and most of the video extracts. Additional resources were also adapted from *Improving Learning in Mathematics*, a government funded program in the UK and from the *Mathematics Assessment Project* funded by the Bill and Melinda Gates Foundation.

Handout 1(1): Sample student work

Interpreting a distance-time graph

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.

Describe what may have happened.
You should include details like how fast he walked.



Maxine's response

when he get out he starts walking fast to the bus stop then he slows down the he picks up the speed again and then ~~the~~ this speed goes ~~cont~~ constant.

What does Maxine appear to understand? How do you know?

List the errors and difficulties that are revealed by Maxine's response.

Try to identify the thinking that lies behind Maxine's error.

What feedback would you give to Maxine?

Jodie's response

Tom walked along a road for
100metres instead of walking
another 30metres he took a
short cut down an alleyway
which took he 20 mintes
he walked very quickly then
he caught the bus to his
college which took about
50mintes.

What does Jodie appear to understand? How do you know?

List the errors and difficulties that are revealed by Jodie's response.

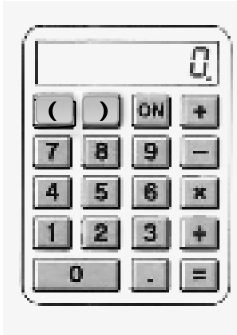
Try to identify the thinking that lies behind Jodie's error.

What feedback would you give to Jodie?

Handout 1(2): Sample student work

Percent changes

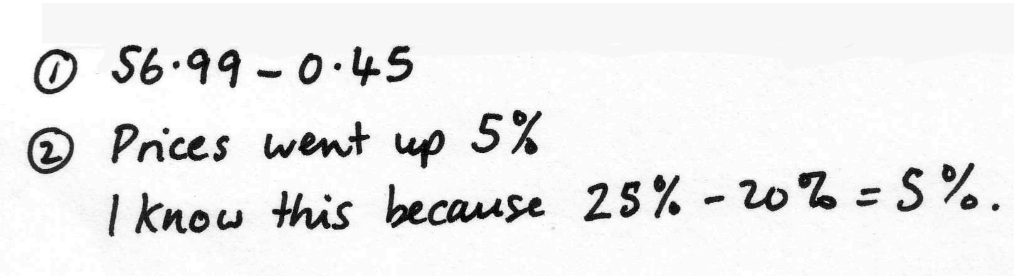
Maria sees a dress in a sale. The dress is normally priced at $56.99.
The ticket says that there is 45% off.
She wants to use her calculator to work out how much the dress will cost.
It does not have a percent button.



Which keys must she press on her calculator?
Write down the keys in the correct order.
(You do not have to do the calculation.)

In a sale, the prices in a shop were all decreased by 20%.
After the sale they were all increased by 25%.
What was the overall effect on the shop prices? Explain how you know.

George's response



① 56·99 − 0·45
② Prices went up 5%
I know this because 25% − 20% = 5%.

What does George appear to understand? How do you know?

List the errors and difficulties that are revealed by his response.

Try to identify the thinking that lies behind each error.

What feedback would you give George? Write down your comments on his work.

Jurgen's response

1. $56.99 \div 100 \times 45 =$

~~56.99 – Ans =~~

$56.99 - 56.99 \div 100 \times 45 =$

2. $\$56.99 = 100\%$

$1\% = 56.99 \div 100 = 0.5699$

$20\% = 0.5699 \times 20 = 11.398$

$25\% = 0.5699 \times 25 = 14.2475$

Difference $= 2.8495$

$\$2.85$

What does Jurgen appear to understand? How do you know?

List the errors and difficulties that are revealed by his response.

Try to identify the thinking that lies behind each error.

What feedback would you give Jurgen? Write down your comments on his work.

Handout 1(3): Sample student work

Interpreting expressions

Britney's response

Interpreting Expressions

1. Write algebraic expressions for each of the following:

   a. Multiply n by 5 then add 4. $n \times 5 + 4$

   b. Add 4 to n then multiply your answer by 5. $n + 4 \times 5$

   c. Add 4 to n then divide your answer by 5. $n + 4 \div 5$

   d. Multiply n by n then multiply your answer by 3. $n \times n \times 3$

   e. Multiply n by 3 then square your answer. $n \times 3^2$

2. Imagine you are a teacher. Decide whether the following work is correct or incorrect.
   If you see an error:

   a. Cross it out and replace it with a correct answer.

   b. Explain the error using words or diagrams.

$2(n+3) = 2n+3$ ✓ Correct

$\frac{10n-5}{5} = 2n-1$? Dont know

$(5n)^2 = 5n^2$ ✓ Correct

$(n+3)^2 = n^2 + 3^2 = n^2 + 9$? Dont know.

What does Britney appear to understand? How do you know?

List the errors and difficulties that are revealed by her response.

Try to identify the thinking that lies behind each error.

What feedback would you give Britney? Write down your comments on her work.

Handout 2(1): Sample follow-up questions

Distance-time graphs: Common issues

Common issues:	Suggested questions and prompts:
Student interprets the graph as a picture For example: The student assumes that as the graph goes up and down, Tom's path is going up and down. Or: The student assumes that a straight line on a graph means that the motion is along a straight path. Or: The student thinks the negative slope means Tom has taken a detour.	• *If a person walked in a circle around their home, what would the graph look like?* • *If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like?* • *In each section of his journey, is Tom's speed steady or is it changing? How do you know?* • *How can you figure out Tom's speed in each section of the journey?*
Student interprets graph as speed–time The student has interpreted a positive slope as speeding up and a negative slope as slowing down.	• *If a person walked for a mile at a steady speed, away from home, then turned around and walked back home at the same steady speed, what would the graph look like?* • *How does the distance change during the second section of Tom's journey? What does this mean?* • *How does the distance change during the last section of Tom's journey? What does this mean?* • *How can you tell if Tom is travelling away from or towards home?*
Student fails to mention distance or time For example: The student has not mentioned how far away from home Tom has travelled at the end of each section. Or: The student has not mentioned the time for each section of the journey.	• *Can you provide more information about how far Tom has travelled during different sections of his journey?* • *Can you provide more information about how much time Tom takes during different sections of his journey?*
Student fails to calculate and represent speed For example: The student has not worked out the speed of some/all sections of the journey. Or: The student has written the speed for a section as the distance covered in the time taken, such as "20 metres in 10 seconds."	• *Can you provide information about Tom's speed for all sections of his journey?* • *Can you write his speed as metres per second?*
Student misinterprets the scale For example: When working out the distance, the student has incorrectly interpreted the vertical scale as going up in tens rather than twenties.	• *What is the scale on the vertical axis?*
Student adds little explanation as to why the graph is or is not realistic	• *What is the total distance Tom covers? Is this realistic for the time taken? Why?/Why not?* • *Is Tom's fastest speed realistic? Is Tom's slowest speed realistic? Why?/Why not?*

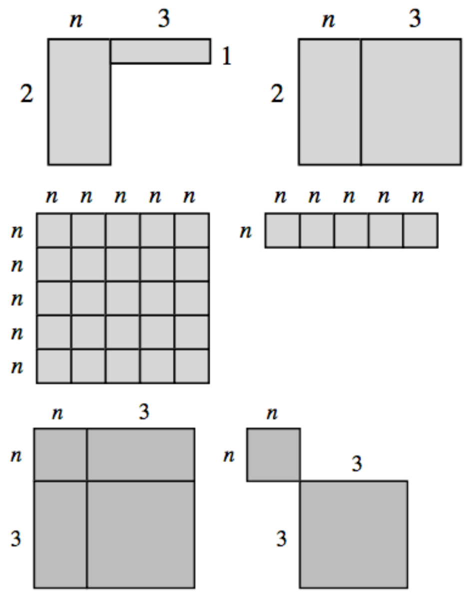
Handout 2(2): Sample follow-up questions

Percent changes: Common issues

Common issues:	Suggested questions and prompts:
Student makes the incorrect assumption that a percentage decrease means the calculation must include a subtraction For example: 56.99 × 0.45 or 56.99 × 1.45 *A single multiplication by 0.55 is enough.*	• *Does your answer make sense? Can you check that it is correct?* • *In a scale, an item is marked "50% off."* • *What does this mean? Describe in words how you calculate the price of an item in the scale. Give me an example.* • *Can you express the decrease as a single multiplication?*
Student converts the percentage to a decimal incorrectly For example: 40.85 + 0.6	• *How can you write 50% as a decimal?* • *How can you write 5% as a decimal?*
Student uses inefficient method For example: First the student calculates 1% , then multiplies by 6 to find 6%, and then adds this answer on: Or: 56.99 × 0.45 = ANS, then 56.99 *A single multiplication is enough.*	• *Can you think of a method that reduces the number of calculator key presses?* • *How can you show your calculation with just one step?*
Student subtracts percentages For example: 25 − 20 = 5%. *Because we are combining multipliers:* *0.8 * 1.25 = 1, there is no overall change in prices.*	• *Make up the price of an item and check to see if your answer is correct.*
Student misinterprets what needs to be included the answer For example: The answer is just operator symbols.	• *If you just entered these symbols into your calculator would you get the correct answer?*

Handout 2(3): Sample follow-up questions

Interpreting expressions: Common issues

Common issues:	Suggested questions and prompts:
Student writes expressions left to right, showing little understanding of the order of operations implied by the symbolic representation. For example: Q1a Writes n × 5 + 4 (not incorrect) Q1b Writes 4 + n × 5 Q1c Writes 4 + n ÷ 5 Q1d Writes n × n × 3	• *Can write answers to the following?* *4 + 1 × 5* *4 + 2 × 5* *4 + 3 × 5* • *Check your answers with your calculator.* • *How is your calculator working these out?* • *So what does 4 + n × 5 mean?*
Student does not construct parentheses correctly or expands them incorrectly. Q1b Writes 4 + n × 5 instead of 5(n + 4) Q1c Writes 4 + n ÷ 5 instead of $\frac{1+n}{5}$ Q2 2(n + 3) = 2n +3 is counted as correct. Q2 $(5n)^2 = 5n^2$ is counted as correct. Q2 $(n+3)^2 = n^2 + 3^2$ is counted as correct.	• *Which of the following is the odd one out and why?* • *Think of a number, add 3, and then multiply your answer by 2.* • *Think of a number, multiply it by 2, and then add 3.* • *Think of a number, multiply it by 2, and then add 6.*
Student identifies errors but does not give explanations. In question 2, there are corrections to the first, third, and fourth statements, but no explanation or diagram is used to explain why they are incorrect.	• *How would you write down expressions for these areas?* • *Can you do this in different ways?* 

Handout 3: Generalisations commonly made by students

Many students’ misconceptions originate from generalising a mathematical concept to a new context in which that concept does not hold. The following list provides examples of such generalisations.

0.567 > 0.85
The more digits a number has, the larger is its value.

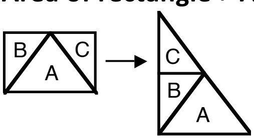
0.4 > 0.62
The fewer the number of digits after the decimal point, the larger is its value. It's like fractions.

3 ÷ 6 = 2
You always divide the larger number by the smaller one.

5.62 x 0.65 > 5.62
Multiplication always makes numbers bigger.

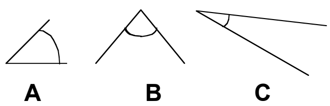
1 litre costs £2.60; 4.2 litres cost £2.60 x 4.2; 0.22 litres cost £2.60 ÷ 0.22
Multiply when the number is bigger and divide when the number is smaller.

Area of rectangle ≠ Area of triangle.



If you dissect and rearrange the pieces, you change the area.

Angle A is greatest. Angle C is greatest.

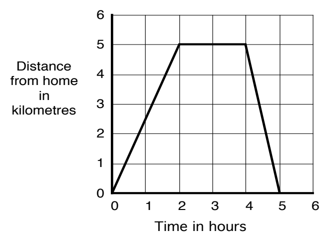


The size of an angle is related to the size of the arc or the length of the lines.

If x + 4 < 10, then x = 5.
Letters represent particular numbers.

3 + 4 = 7 + 2 = 9 + 5 = 14
Equals' means 'makes'.

Graphs are just pictures.



He was going up a steep hill, across the top and down the other side.

What other examples can you add to this list?

Can you think of any misconceptions you have had at some time? How were these overcome?

Handout 4: Handling students' errors

There are two common ways of reacting to pupils' errors and misconceptions:

- **Avoid them** whenever possible:
  "If I warn pupils about the misconceptions as I teach, they are less likely to happen. Prevention is better than cure."
- **Use them** as learning opportunities:
  "I actively encourage students not to hide mistakes when they make them, and to learn from them."

Which approach resonates with your own practice?

What kind of challenges are there to using students' mistakes in a classroom?

Handout 5: Principles to discuss

The following principles are backed up by research evidence:

Explore misconceptions through discussion

Teaching approaches that encourage the exploration of misconceptions through discussions result in deeper, longer-term learning than approaches that try to avoid mistakes by explaining the 'right way' to see things from the start.

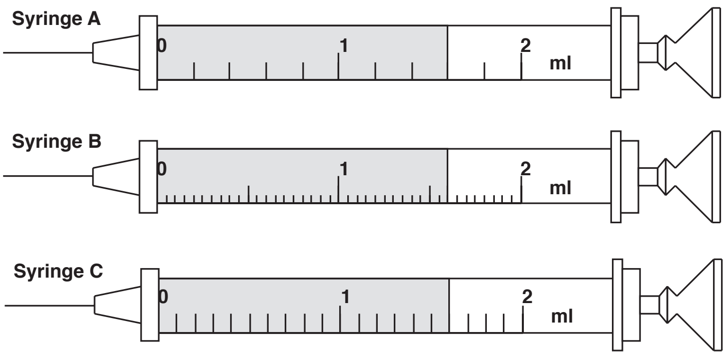
Focus on known difficulties

It is helpful if discussions focus on known difficulties. Rather than posing many questions in one session, it is better to focus on a challenging question and encourage a variety of interpretations to emerge, so that learners can compare and evaluate their ideas.

Use cognitive conflicts

Questions can be juxtaposed in ways that create a tension (sometimes called a 'cognitive conflict') that needs resolving. Contradictions arising from conflicting methods or opinions can create awareness that something needs to be learned.

> For example, asking learners to say how much medicine is in each of the following syringes may result in answers such as "1.3ml, 1.12ml and 1.6ml". "But these quantities are all the same!" This provides the start for a useful discussion on the denary nature of decimal notation.



Provide opportunities for meaningful feedback

Activities should provide opportunities for meaningful feedback. This does not mean providing summative information, such as the number of correct or incorrect answers. More helpful feedback is provided when learners compare results obtained from alternative methods until they realise *why* they get different answers.

Include whole group discussions

Sessions include time for whole group discussion in which new ideas and concepts are allowed to emerge. This requires sensitivity so that learners are encouraged to share tentative ideas in a non-threatening environment.

Allow opportunities to consolidate

Opportunities should be provided for learners to consolidate what has been learned through the application of the newly constructed concept.

What do you think of this advice?

What would you like to add?

Is there anything you would remove?