FaSMEd

Raising Achievement through Formative Assessment in Science and Mathematics Education

Interpreting equations

Subject: Mathematics
Age of students: 14 - 15 years

Technology: Card

Functionalities: Sending and displaying

Time: 1 or 2 lessons

FaSMEd partner: African Institute for Mathematical Sciences Schools Enrichment Centre

Short Abstract: This lesson is about interpreting algebraic equations and relating them to real-life situations. After a short introduction, during which the teacher leads a discussion about variables, the students work in small groups. They are told what each of four variables (*a, b, x* and *y*) represents. They match two sets of small cards: one with equations, such as *b=2*, and the other with situations, such as 'Bananas cost R2'. After the activity the teacher holds a class discussion aiming to go over the answers and draw out something about the students' own learning and to think about the bigger conceptual issues.

1. Content

The lesson is about reinforcing students' understanding of algebraic equations and relating equations to descriptions of situations.

2. Activity

2.1 Aims

This lesson is intended to help students:

- Connect algebraic equations to real-life situations.
- Uncover and address misconceptions concerning the meaning of variables in equations.

The two main ideas in the lesson are that if $y = 4x$ then y is bigger than x and that it is very important to know what a variable refers to.

2.2 Structure / Methodology

Time needed: 60 to 90 minutes

Materials needed:

- Mini whiteboards, wipes and pens
- Poster paper and glue or similar
- Sets of small cards for students to match (see Figure 3, 4 and 5)
- Sets of big cards for use in the whole-class discussion
- Powerpoint presentation (optional, see Figure 1 and 2)

2.3 Lesson plan

This is an abbreviated lesson plan, adapted from the one found at http://map.mathshell.org/lessons.php?unit=6215&collection=8.

Introduction

Revise the notion of variables in algebra with the learners.

Hand out a mini white board (or a blank piece of paper) to each learner (or pair of learners). Write the equation *x* = 4*y* on the board, or display the first slide of the PowerPoint presentation if you are using it. Ask them to discuss in pairs which is bigger: *x* or *y*. If you think further whole class discussion is needed, continue with other slides in the PowerPoint, if you are using it; otherwise ask them to make an equation linking *e* and *b* when *e* is the number of eggs and *b* is the number of eggs boxes, each holding six eggs (see Figure 1).

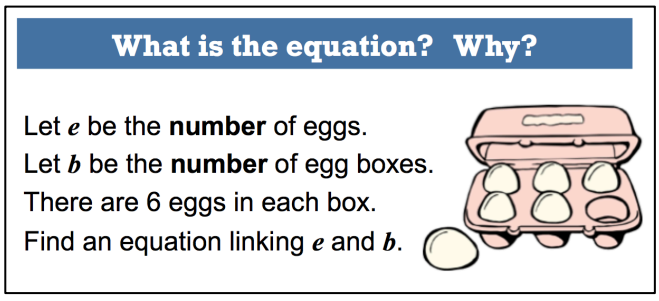


Figure 1: Make an equation (1)

Go on to ask them the questions in Figure 2, below. Note that this question relates to the **cost** of eggs and egg boxes, rather than the **number** of eggs and egg boxes, as was the case previously.

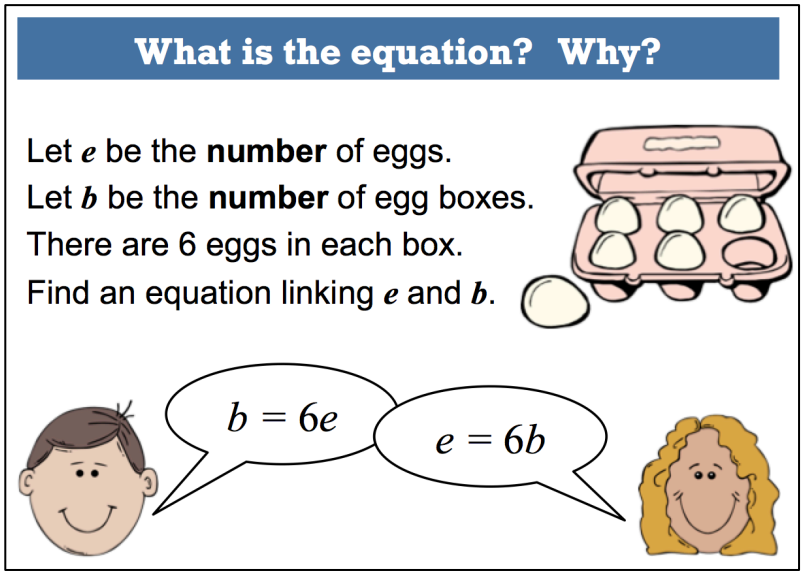


Figure 2: Make an equation (2)

Card matching activity

Put the learners into pairs. Each group should have:

- the definition of the variables (See Figure 3)
- a set of expression cards (E1-E12) (See Figure 4)
- a set of solution cards (S1-S10) (See Figure 5)
- a large sheet of paper
- glue or Prestik/blutack.

Explain to them that they should match equation cards with statement cards. You might like to point out that it is not a one-to-one match and they should end up with nine groups of cards.

a = the **number** of apples I bought	x = the **cost** of an apple in Rand
b = the **number** of bananas I bought	y = the **cost** of a banana in Rand

Figure 3: Definition of variables

E1 $y = 2x$	E7 $b = 2$
E2 $x = \frac{1}{2}y$	E8 $x = 2y$
E3 $b = 2a$	E10 $x = y$
E4 $a = \frac{1}{2}b$	E9 $y = \frac{1}{2}x$
E5 $b + a = 25$	E11 $ax + by = 25$
E6 $ax = 25$	E12 $y = 2$

Figure 4: Equation cards

S1 Apples cost half as much as bananas.	S2 Bananas cost twice as much as apples.
S3 I bought twice as many bananas as apples.	S4 Altogether I bought 25 apples and bananas.
S5 Altogether the apples I bought cost R25.	S6 A banana costs R2.
S7 Apples and bananas cost the same.	S8 I paid R25 for all the apples and bananas I bought.
S9 Bananas cost half as much as apples.	S10 I bought 2 bananas.

Figure 5: Statement cards

Allow the learners enough time to do the activity.

Try to avoid telling them the answers. Listen to what they are saying and ask them questions. You may need to remind them that *a* and *b* represent numbers of apples and bananas and *x* and *y* represent costs.

Discussion

Have all the big cards on a table at the front of the class. Ask learners to come up and find matching cards and to stick these on the board. Alternatively hand out the big 'equation' cards to specific learners and ask them to find matching 'statement' cards and to stick them on the board.

2.4 Technology

- Mini whiteboards for use in the introduction and during the lesson.
- Sets of small cards for use by the small groups of students.
- Sets of big cards for use in the whole class discussion.

All these technologies provide opportunities for formative assessment in that they help to make the students' thinking visible.

2.5 Aspects of Formative Assessment

The teachers assess the students' levels of understanding in real-time and act accordingly by asking questions, giving hints and sometimes telling them the answer. They clarify learning intentions, engineer effective discussions, provide feedback and activate students as resources for one another and for themselves. This takes place at all phases of the lesson.

In the main card-matching activity, students work in pairs to match cards which represent the same thing. The intention is that students will take turns to select and match cards and will explain their thinking to the others in their group. Each student should engage with their peer's explanation and take responsibility for the understanding of others in their group.

If the students follow these instructions, it is likely that they will be assessing their peers' levels of understanding and acting accordingly, sometimes explaining their own reasoning, sometimes telling the peers what to do and sometimes asking questions (i.e. peer formative assessment). In this case they would be activating their peers as instructional resources.

3. Further Information

3.1 Background

This lesson is based on the one designed by the Mathematics Assessment Project, which can be found at http://map.mathshell.org/lessons.php?unit=6215&collection=8.

The lesson design includes the use of a pre-lesson assessment task, which teachers can use to gather information about the current levels of students' understanding and their different problem solving approaches. The use of this task is highly recommended but in practice many teachers do not have time to do so. A post-lesson task is also provided.

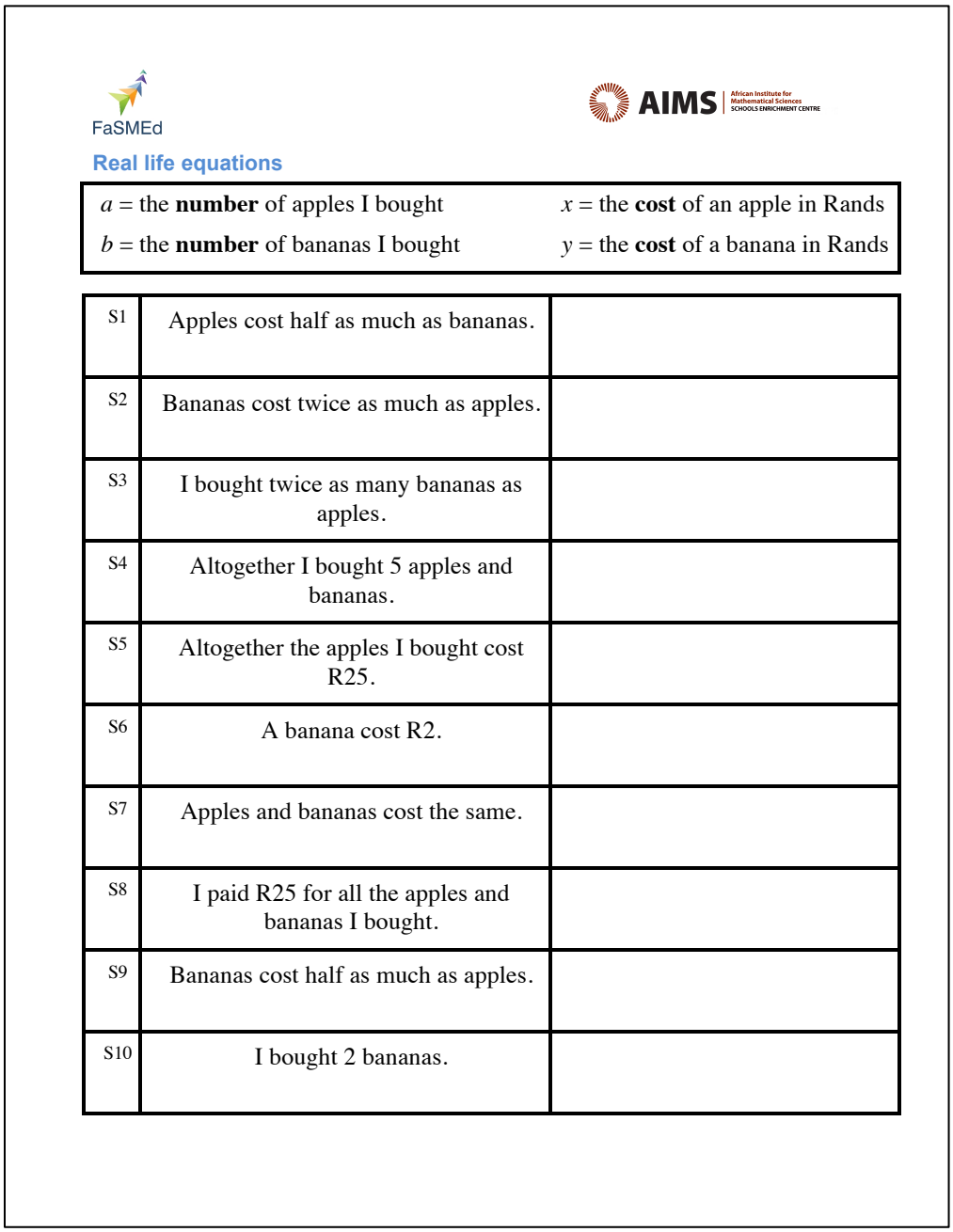
3.2 Common issues

The issues arising in the pre-lesson task are likely to arise in the main card matching activity and the table of common issues below provides some useful questions and prompts teachers could use to move the students' thinking on.

Common issues	Suggested questions and prompts
Q1. Student selects the wrong equations For example: The student selects the equation $y=4x$.	• Which is the greater number: the number of chairs or the number of legs? • Say the equation in words.
Q1. Student selects only one of the two correct equations	• Is this the only equation that is correct?
Q1. Student selects equations that are incompatible. For example: The student selects the first two equations.	• Suppose that $y=12$ (there are 12 legs). What do these two equations say about the number of chairs? Which one makes sense?
Student misinterprets the meaning of the symbols. For example, the student interprets the variables as if they were the objects. Q1. The student selects the first equation $x = 4y$ and writes that it means 'Chair has four legs'. Q2. For $p = 2e$ student writes 'There are two erasers for every pen'.	• Suppose that $y=12$ (there are 12 legs). What does your equation say about the number of chairs? • Which is greater, x or y? Why? • What does p stand for? What does e stand for?
Student substitutes the phrases literally, but does not interpret the meaning in context. For example: Q2. The student writes 'Two times e equals p' or 'x equals two times y'.	• What do you know about the costs of the pens and erasers? • What does $2p$ represent? • Can you write the meaning as you would say it in everyday life?
Student answers both questions correctly.	• Can you write an equation which represents the total amount Max spent?

3.3 Answers

Many teachers like students to keep a record of their work from this lesson in their books. Figure 6 below shows a record sheet designed for use by students.



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Real life equations

a = the **number** of apples I bought	x = the **cost** of an apple in Rands
b = the **number** of bananas I bought	y = the **cost** of a banana in Rands

S1	Apples cost half as much as bananas.	
S2	Bananas cost twice as much as apples.	
S3	I bought twice as many bananas as apples.	
S4	Altogether I bought 5 apples and bananas.	
S5	Altogether the apples I bought cost R25.	
S6	A banana cost R2.	
S7	Apples and bananas cost the same.	
S8	I paid R25 for all the apples and bananas I bought.	
S9	Bananas cost half as much as apples.	
S10	I bought 2 bananas.	

Figure 6: Learner recording sheet

The answers to the matching task can be found in the full lesson notes and are also provided below (Figure 7).

S1 and S2	Apples cost half as much as bananas. Bananas cost twice as much as apples.	$x = \frac{1}{2}y$; $x = 2y$
S3	I bought twice as many bananas as apples.	$b = 2a$
S4	Altogether I bought 5 apples and bananas.	$b + a = 5$
S5	Altogether the apples I bought cost R25.	$ax = 25$
S6	A banana costs R2.	$y = 2$
S7	Apples and bananas cost the same.	$x = y$
S8	I paid R25 for all the apples and bananas I bought.	$ax + by = 25$
S9	Bananas cost half as much as apples.	$x = \frac{1}{2}y$ $y = 2x$
S10	I bought 2 bananas.	$b = 2$

Figure 7: Answers to matching task