FaSMEd

Raising Achievement through
Formative Assessment
in Science and Mathematics
Education

Can I sketch a graph based on a given situation? – A digital tool for formative self-assessment

Subject:	Mathematics
Age of students:	14 - 18 years
Hardware:	iPads (or Laptops, or Pcs)
Software:	TI-Nspire CAS Application
Functionalities:	Providing an Interactive Environment
Time:	60 minutes
FaSMEd partner:	University of Duisburg-Essen

Short Abstract:

Using the tool, students assess their own competencies rather than having the technology evaluate them. Thus, learners are provided with a check-list that implies information on typical misconceptions in the field of functions to examine their own solution to an open assessment task. Finally, they can work on various information units, practice or expand tasks according to their individual needs.

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1. Content

The digital self-assessment tool covers the mathematical topic of graphs and functions. Relations between two quantities and their representations are in focus. Especially the translation of a situational description of a function into its graph is being addressed.

2. Activity

2.1 Aims

The digital self-assessment tool helps students to assess how well they are able to perform the translation between a situational description of a functional relation into a graph. The ability to translate between such forms of representation flexibly is a key aspect of understanding the concept of functions (Duval 2002).

When using a graph to describe a real situation, students not only have to consider different types of representation, but various aspects of functions: mapping, covariation and object (Vollrath 1989). The mapping aspect focuses on a function on a static and local level. One value of the independent quantity is assigned to exactly one value of the dependent quantity. Therefore, when viewing the graph of a function, single points as representations of ordered pairs of values are in focus. The aspect of covariation demands a more global and dynamic perception of a functional relation (Malle 2000). This aspect focuses on the variation of the quantities' values with each other: How does the value of one quantity change when the other one is altered? Finally, a function can be viewed on a global level. Taking all pairs of values into account as a whole, leads to considering the function as a new object (Vollrath 1989). The development of these three basic concepts (in German: *Grundvorstellungen*) is essential for building a mathematical understanding of functions because they are the link between the mathematical concept and a real situation (vom Hofe 2003; Leuders & Prediger 2005). This is why, the digital self-assessment tool allows students to explore all three aspects of functions.

The tool assesses and supports the following general mathematical competencies, that are defined in the German national educational standards for an intermediate school-leaving qualification (KMK, 2004):

The student is able to:

- model real life situations mathematically,
- use mathematical representations.

Furthermore, the following content-related competencies of the German national educational standards regarding the concept of functions are assessed and supported:

The student is able to:

- use functions to describe quantitative relations,
- recognize and describe functional relations as well as represent these verbally or graphically,
- analyse, interpret and compare different representations of functional relations (such as linear, proportional and inversely proportional),
- describe variations of quantities using functions.

2.2 Structure / Methodology

The digital self-assessment tool is designed to allow the students to become their own assessors rather than having the technology evaluate their answers. Therefore, students are provided with a check-list that implies information on typical misconceptions in the field of functions and graphs to examine their answer to an open assessment task. Based on this self-assessment, learners can choose to view more information on their previous mistakes and work on specific practice tasks.

The tool is suitable for students from age 14 (grade 7/8, when they are first introduced to the topic of functions and graphs) until the upper secondary level (age 18). It can be used whenever the teacher or student feels the need to repeat this topic and assess the student's competencies of sketching a graph based on a given situation. It is not intended to be used in order to introduce or first learn about the topic.

The student works individually and assesses his/her own competencies. Furthermore, the learner decides on his/her own which information to view and which practice tasks to work on based on the *check*.

It takes approximately 45-60 minutes to work through the digital self-assessment tool.

The digital self-assessment tool includes five components, namely:

- *Test* (labelled with a magnifying glass),
- *Check* (labelled with a check marks)
- *Info* (labelled with a light bulb),
- *Practice* (labelled with a notebook),
- *Expand* (labelled with gearwheels).

The hyperlink structure of the digital self-assessment tool is shown in figure 1:

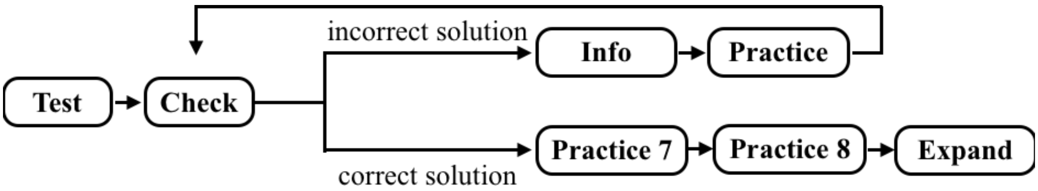

Figure 1: Hyperlink structure of the digital self-assessment tool

A student begins by working on the *test* task (figure 2). The learner is given a story about a bike ride without specific values for the speed and is asked to sketch a graph that shows how the speed changes as a function of the time. Then the student looks at a sample solution including a list of criteria for successfully solving this task, before moving on to the *check*. The *check* consists of six statements (e.g. "I realized when the graph is increasing, decreasing, or remains constant.") that help the learner to reflect on their own solution and identify their mistakes. For each statement, the student decides weather it is true or false for his/her own graph (figure 2).

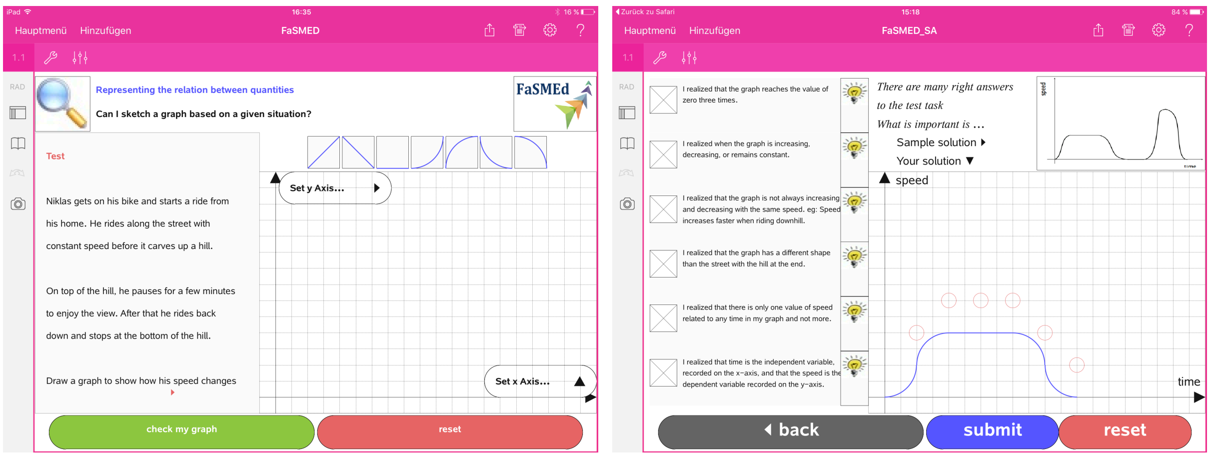

Figure 2: *Test* and *Check* of the digital self-assessment tool

If an error was identified, the learner clicks on the "lightning bulb" button next to the check-point and is referred to an *Info* on that particular mistake (figure 3). A matching *practice* task gives the opportunity to test one's understanding on the renewed information. Once again, the student assesses his/her own competencies by comparing his/her own to the given solution. This way, the tool encourages the learner to reflect on his/her work rather than generating a feedback for him/her. Afterwards, the student goes back to the *check* and continues with the next check-point.

If the sketched graph is stated as correct by the learner by marking off all check-points, he/she can work on two more *practice* and finally an *expand* task with a more complex context.

The digital self-assessment tool includes four different task types:

- *Graphing*: Students sketch a graph by dragging movable and adjustable graph-tiles in-to a graphing window. The axes of the coordinate system are labelled by choosing between different options from a drop-down menu. (Used for these tasks: Test, Practice 3 part 1, Practice 6, Practice 7 part 2, Practice 8 and Expand, see figure 2)
- *Open answer*: Students can type in an answer into a text box. (Used for these tasks: Practice 3 part 2 and 3, see figure 3)
- *Selection*: Students answer by selecting situations for which a certain question is true by double-clicking on buttons that match the situations numbered labels. (Used for these tasks: Practice 1 and Practice 5, see figure 4)
- *Matching*: Students match situations with graphs by first selecting a situation when they click on the button matching the situation's number and then clicking on a graph. (Used for these tasks: Practice 2, Practice 4 part 1 and Practice 7 part 1, see figure 4)

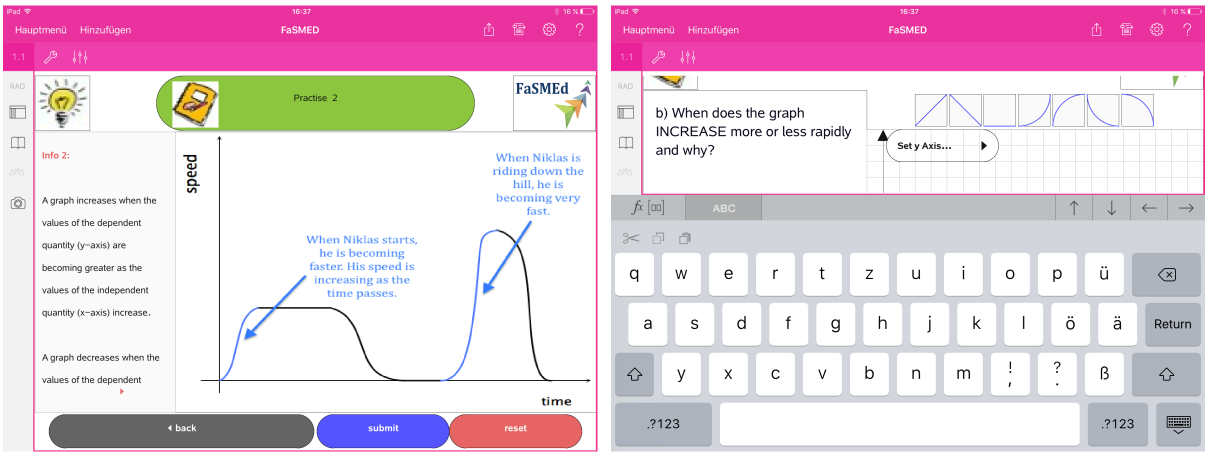

Figure 3: Examples for an *Info* and an *open answer task* of the tool

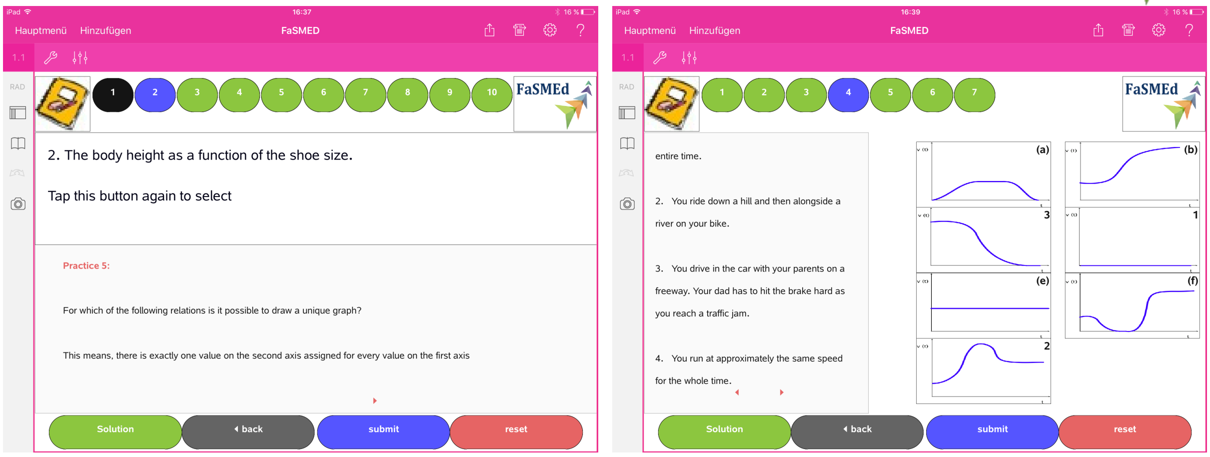

Figure 4: Examples for a *selection* and a *matching* task of the tool

For more explanations on the digital self-assessment tool including video tutorials, please visit http://compasstech.com.au/FaSMED/.

2.3 Technology

The digital tool for formative self-assessment is run in the software TI-Nspire CAS by Texas Instruments. Due to the big touch screen, it is best to be used on iPads, but also works on Computers and Handhelds. Once the application is installed, the tool's file can be accessed via the application. (A link to download the application is included in section 4. References of this document.)

The tool gives the students the opportunity to explore the mathematical content of functions and graph by using dynamic representations (graphing tasks) as well as offering information on typical misconceptions in form of the *check*. This is why, the functionality of the technology can be classified as ***Providing an Interactive Environment***.

2.4 Aspects of Formative Assessment

As ***students*** work on the tool individually and are the active agents of the assessment, we can, thus, focus on the formative assessment (FA) strategies the learners use.

Within the interactive environment of the digital self-assessment tool, learners can use four of the FA strategies of the FaSMEd Framework (according to Wiliam & Thompson 2007): First, they are able to ***understand the learning intentions*** by being presented with the question "Can I sketch a graph based on a given situation?" What is more, the *check* provides them with information on typical misconceptions. This gives students the possibility to recognize ***criteria for success*** in terms of solving the *test* task. Furthermore, the students work on different tasks and need to compare their own solutions to a sample solution. This is how they ***elicit evidence about their own understanding***. After solving a task or thinking about a certain check-point, the students need to decide themselves, which step to take next in order to move their learning forward. Therefore, the learner is encouraged in ***providing (self-) feedback that moves his/her learning forward***. Finally, the student is ***activated as the owner of his/her own learning*** in the sense that he/she is challenged to think about their own learning on a metacognitive level as the tool does not advise them on which mistakes were made or what steps to take next. The learner investigates his/her own (mis-)conceptions and adopts responsibility for his/her learning.

Thus, referring to the FaSMEd Framework (see the FA in Theory section of the FaSMEd toolkit for more information), we can highlight the following four cuboids in order to characterise the possible FA processes when working with the digital self-assessment tool:

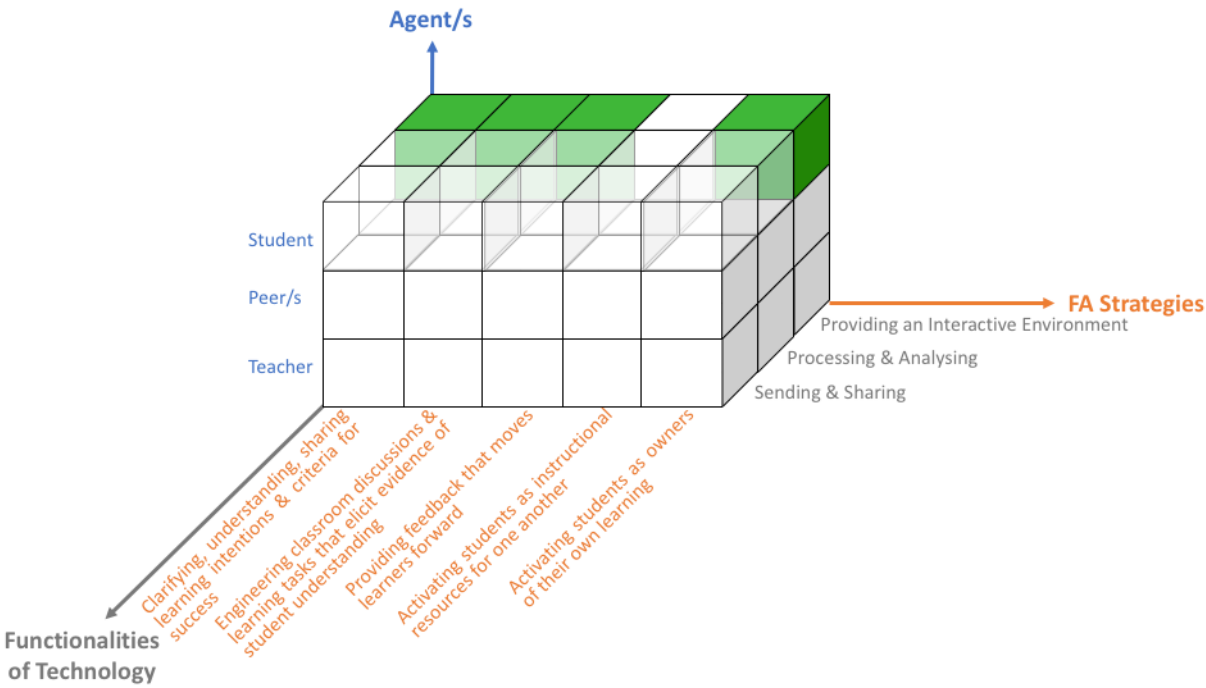

Figure 5: Possible FA strategies used by students working with the digital self-assessment tool functioning as an interactive environment

3. Further Information

3.1 More features of the digital self-assessment tool

A language selection between German and English was introduced to the digital self-assessment tool in order to make its use easy in German schools as well as for FaSMEd partners. Moreover, by inserting *submit* buttons for every exercise of the tool, the software is able to store student answers and copy it into a class list in the form of an Excel spread sheet in google drive for the teacher to review his/her students' work. Steve Arnold from Texas Instruments set up a webpage to instruct teachers on how to generate this document and introduce the digital self-assessment tool to them via written explanations and video tutorials (see http://compasstech.com.au/FaSMED/).

3.2 Pen-and-paper version of the tool

The pen-and-version of the tool is an alternative to the digital version when the required software can not be provided. It consists of seventeen cards and is structured in the same way as the digital tool. It entails the same tasks, check-list and information. In comparison to the digital tool, the pen-and-paper version does not provide dynamic representations and requires more organisational skills of the students as the tools' hyperlink structure can not be followed as intuitively.

The pen-and-paper version can be downloaded from the FaSMEd toolkit.

3.3 Using the tool in a classroom discussion

Although the tool is designed for self-assessment, there are other methods to purposefully use the tool in a classroom. As the student's work can be captured by the tool and shown to the teacher in a google drive spread sheet, the teacher has the opportunity to assess the learners' understanding after their self-assessment. Therefore, making it possible for the teacher to address further difficulties in upcoming lessons. Moreover, the tool offers the same structure as an assessment lesson (for example the "interpreting distance-time graphs" lesson of the MARS project). The *test* task functions as the initial assessment task and the *check*-list includes possible issues that students might have with the content. Thereby making it easy for teachers to adjust the tool to be used in a teacher or peer-assessment lesson. Table 1 suggests questions and prompts to encounter the students' mistakes and difficulties in a classroom discussion:

<table>
<tr><th>Common issues</th><th>Suggested questions and prompts</th></tr>
<tr><td>The student doesn't realize when the graph reaches the value of zero.

Can be recognized when:

The student draws a graph that does not start at the point of origin.

The student draws a graph that does not reach the value of zero three times.</td><td>Where does your graph start? Why?

When does your graph touch the x-axis?

Can you tell me the story of Niklas’ bike ride using your graph?

Which quantity is recorded on the y-axis? Which value does it reach at the very beginning, when Niklas stops on top of the hill, at the very end?

When does Niklas’ speed reach a value of 0 km/h. What does this mean for your graph?

When does a graph reach the value of zero? Is this the case in the given situation?</td></tr>
<tr><td>The student is unable to recognize and illustrate the graph's slope.

Can be recognized when:

The student draws a graph that doesn't increase to illustrate that Niklas is getting faster on his bike.

The student draws a graph that doesn't decrease to illustrate that Niklas is slowing down.

The student draws a graph that doesn’t remain constant to illustrate that Niklas’ speed does not change over a period of time.</td><td>When does Niklas’ speed increase/decrease/remain constant? What does this mean for your graph?

When is Niklas getting faster/slower/driving with constant speed? What does this mean for your graph?

What does your graph look like when Niklas is getting faster/slower/drives with a constant speed? At what times is this the case in the given situation?

When does your graph increase/decrease/remain constant? Why?

Can you tell me the story of Niklas’ bike ride using your graph?

How is Niklas’ speed changing in this period of time according to your graph?</td></tr>
<tr><td>The student is unable to recognize the steepness of the graph’s slope.

Can be recognized when:</td><td>Is your graph always increasing/decreasing with the same speed? Why?

Is Niklas’ speed changing by the same amount at all times?</td></tr>
</table>

The student draws a graph in which the slope does not change. The student draws a graph with the same slope to illustrate that Niklas is getting faster when he rides along the street and when he rides downhill.	What does this mean for the graph? Is Niklas' speed increasing faster when Niklas rides downhill compared to when he starts his bike ride? When is the speed reaching its maximum? What does this mean for your graph? When does the speed change more or less rapidly? What does this mean for your graph?
The student interprets the graph as a picture. *Can be recognized when:* The student draws "a street with a hill at the end" as the time-speed-graph.	Picture Niklas' ride at different times. What is his speed at each of those moments? What might be Niklas' speed approximately when he rides along the street/rides up the hill/rides down the hill/etc.? How much time has passed until then? Which point in the coordinate system could therefore be part of your graph? What could Niklas' speed be after 5/10/15 minutes? What does this mean for your graph? Choose one point of your graph. In the given situation, interpret the meaning of your graph running through this point. Is that possible? How does Niklas' speed change as a function of the time in each section of his ride? What does this mean for your graph? What would your graph look like, if Niklas was riding with a constant speed for the whole time?
The student disregards the uniqueness of the function. *Can be recognized when:* The student assigns several values of speed to one single value of time. The student's graph is non-unique.	How many different values of speed is Niklas able to reach at each moment during his ride? What does this mean for your graph? Is it possible for Niklas to have more than one speed at a time? What does this mean for your graph? What is your graph representing? Does it have to be unique? Is it possible to assign several values on the second axis to one value on the first axis? Why/Why not?

The student's graph doesn't represent the functional relation between time and speed.	
The student swaps the axes. *Can be recognized when:* The student records the speed on the x-axis and the time on the y-axis.	Which relation is represented in your graph? What can you read off your graph? Which quantity is recorded on the x-axis? Which quantity is recorded on the y-axis? Which one of the described quantities is the dependent/independent quantity? Where is this quantity recorded in the coordinate system? How do you know which quantity you have to record on the x/y-axis?

Table 1: Common issues and questions/prompts to encounter them

4. References

The TI-Nspire Navigator application can be downloaded on the App Store. For more information, see: https://education.ti.com/en-GB/uk/products/apps/ti-nspire-cas-app-for-ipad/tabs/overview

Video tutorials and descriptions on the digital self-assessment tool can be found here: http://compasstech.com.au/FaSMED/

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