



# FaSMEd

Raising Achievement through  
Formative Assessment  
in Science and Mathematics  
Education



## Distance-time graphs

<b>Subject:</b>	Mathematics
<b>Age of students:</b>	14 - 15 years
<b>Used Technology:</b>	Card
<b>Functionalities:</b>	Sending and displaying
<b>Time:</b>	1 or 2 lessons
<b>FaSMEd partner:</b>	African Institute for Mathematical Sciences Schools Enrichment Centre
<b>Short Abstract:</b>	This lesson is about interpreting distance-time graphs. After a brief introduction supported by a PowerPoint presentation, students work in pairs on a card-matching activity. There are three sets of cards: graphs, stories and tables of data. Students are first given the graph and story cards to match, and once they have matched these, they are given the third set of cards, asked to match them with the existing pairs and possibly make changes to the existing pairs. After the activity the teacher holds a class discussion aiming to draw out bigger conceptual issues such as how gradient relates to speed.

## 1. Content

The lesson is about interpreting distance-time graphs. Students need to interpret a distance-time graph in order to match the graph with a descriptive story and a table of data.

## 2. Activity

### 2.1 Aims

This lesson is intended to help teachers assess how well students are able to interpret distance-time graphs and, in particular, to help identify students who:

- Interpret distance-time graphs as if they are pictures of situations rather than abstract representations of them.
- Have difficulty relating speeds to slopes of these graphs.

### 2.2 Structure / Methodology

Time needed: 60 to 90 minutes

Materials needed:

- Matching a Graph to a Story slide (or equivalent) see Figure 1 below
- Mini whiteboards, wipes and pens
- Poster paper and glue
- Sets of small cards for students to match (see Figure 2)
- Sets of big cards for use in the whole-class discussion

### 2.3 Lesson plan

This is an abbreviated lesson plan, adapted from the one found at <http://map.mathshell.org/lessons.php?unit=8225&collection=8>



## Introduction

Hand out a mini whiteboard (or a blank piece of paper) to each learner (or pair of students). Display the graph and three stories in Figure 1 on the board (either by using the PowerPoint slide or by photocopying and enlarging the graph and stories) and ask the students to match the correct story to the graph and to write their answer on their mini whiteboard or piece of paper.

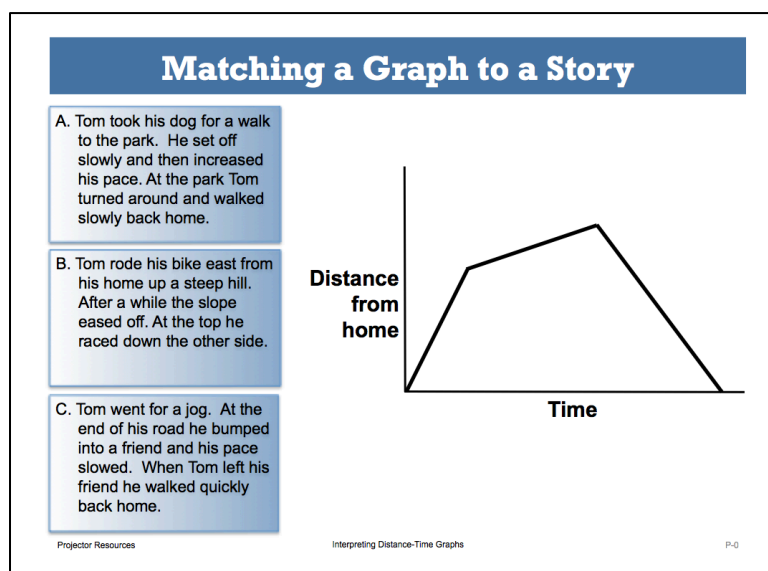
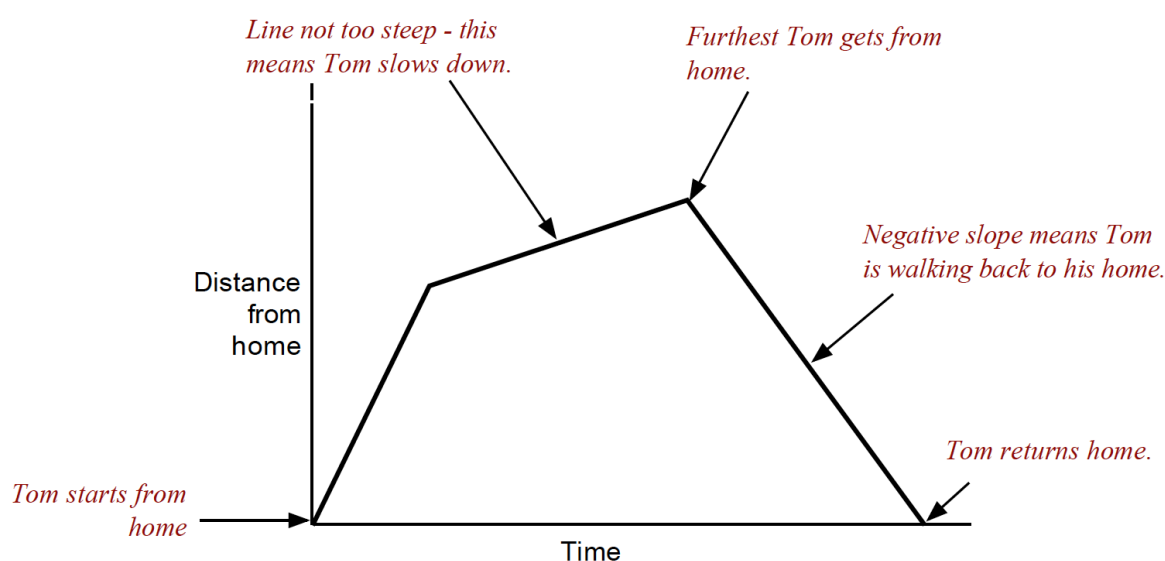


Figure 1: A graph and three stories

Ask the students to show you their answers. Ask one or two students who chose option A to justify their choice. Even if their explanations are incorrect or only partially correct, write them next to the appropriate section of the graph. Encourage the other students to challenge these interpretations.

Repeat this with options B and C.

A graph may end up looking like this:





### Card matching activity

Put the students into pairs. Each pair should have:

- a set of graph cards (A-J, see Figure 2)
- a set of story cards (S1-S10, see Figure 3)
- a set of table cards (T1-T11, see Figure 4)
- a large sheet of paper
- glue or Prestik/bluetack.

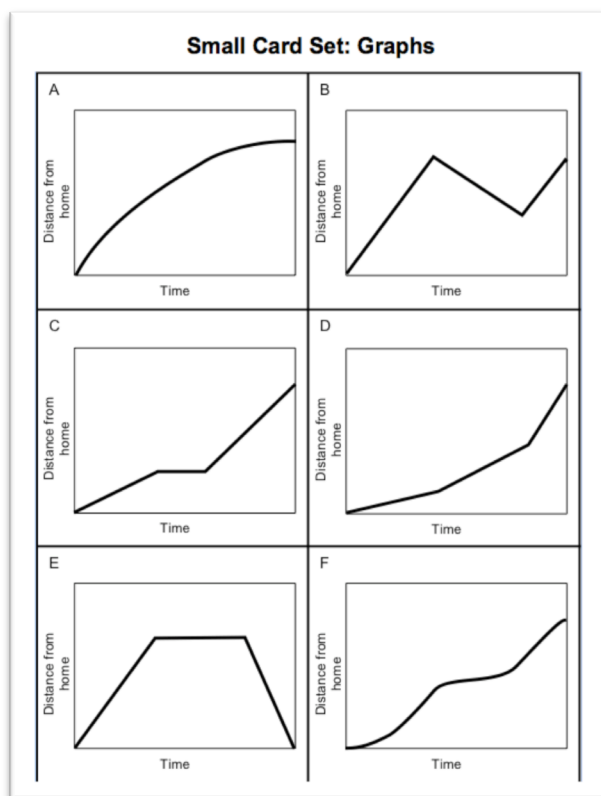


Figure 2: Small graph cards

**Small Card Set: Stories**

1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.
3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	4 Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.
5 Tom left his home for a run, but he was unfit and gradually came to a stop!	6 Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.
7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	8 This graph is just plain wrong. How can Tom be in two places at once?
9 After the party, Tom walked slowly all the way home.	10 Make up your own story!

Figure 3: Small story cards

## Small Card Set: Tables

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Figure 4: Small table cards



Explain that the students should take turns to match a graph card with a story card. Encourage them to explain their thinking carefully and clearly to their partner. They should place their cards side by side on the large sheet of paper, not on top of one another, so that everyone can see them. They should also leave space around the cards as they will eventually be adding the table cards.

You can also share these instructions with the class by using slide 2 of the projector resources.

Some of the cards are deliberate distracters. For example, a learner who matches Card S2 and E (see Figure 5) suggests that they think that graphs are pictures of the situation.

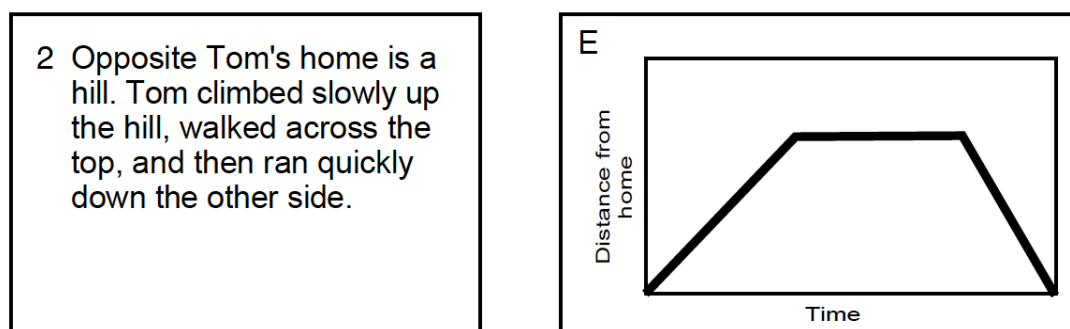


Figure 5: Deliberate distracters

Once students have matched each graph card with a story card hand out the table cards. Explain to the students that they should take turns to match the table cards with the cards that are already on their desk. Remind them to explain their thinking carefully and clearly.

Once they have matched all the cards they should stick them down together on the poster.

Allow the students enough time to do the activity.

Try to avoid telling them the answers. Listen to what they are saying and ask them questions. You may need to ask them about, for example, when Tom is at home or when he is furthest from home.

If some students finish quickly, encourage them to make up their own pairs of cards.

### Discussion

Stick the big versions of the graph cards on the board. Have the big version of the story and table cards on a table at the front of the class. Ask students to come up and find the matching story and/or table graphs for a graph card and to stick these on the board. Alternatively hand out the big story and table cards to specific students and ask them to stick the card on the board under the matching graph card.

## 2.4 Technology

- Mini whiteboards for use in the introduction and during the lesson.
- Sets of small cards for use by the small groups of students
- Sets of big cards for use in the whole class discussion

All these technologies provide opportunities for formative assessment in that they help to make the students' thinking visible.

## 2.5 Aspects of Formative Assessment

The teachers assess the students' levels of understanding in real-time and act accordingly by asking questions, giving hints and sometimes telling them the answer. They clarify learning intentions, engineer effective discussions, provide feedback and activate students as resources for one another and for themselves. This takes place at all phases of the lesson.

In the main card-matching activity, students work in pairs to match cards of equal value. The intention is that students will take turns to select and match cards and will explain their thinking to the others in their group. Each student should engage with their peer's explanation and take responsibility for the understanding of others in their group.

If the students follow these instructions, it is likely that they will be assessing their peers' levels of understanding and acting accordingly, sometimes explaining their own reasoning, sometimes telling the peers what to do and sometimes asking questions (i.e. peer formative assessment). In this case they would be activating their peers as instructional resources.

## 3. Further Information

### 3.1 Background

This lesson is based on the one designed by the Mathematics Assessment Project, which can be found at <http://map.mathshell.org/lessons.php?unit=8225&collection=8>.

The lesson design includes the use of a pre-lesson assessment task, which teachers can use to gather information about the current levels of students' understanding and their different problem solving approaches. The use of this task is highly recommended but in practice many teachers do not have time to do so. A post-lesson task is also provided.

### 3.2 Common issues

The issues arising in the pre-lesson task are likely to arise in the main card matching activity and the table of common issues below provides some useful questions and prompts teachers could use to move the students' thinking on.



## Common issues

## Suggested questions and prompts

<p><b>Student interprets the graph as a picture</b></p> <p>For example: The student assumes that as the graph goes up and down, Tom's path is going up and down.</p> <p>Or The student assumes that a straight line on a graph means that the motion is along a straight path.</p> <p>Or: The student thinks the negative slope means Tom has taken a detour.</p>	<ul style="list-style-type: none"> <li>• <i>If a person walked in a circle around their home, what would the graph look like?</i></li> <li>• <i>If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like?</i></li> <li>• <i>In each section of his journey, is Tom's speed steady or is it changing? How do you know?</i></li> <li>• <i>How can you figure out Tom's speed in each section of the journey?</i></li> </ul>
<p><b>Student interprets graph as speed-time</b></p> <p>The student has interpreted a positive slope as speeding up and a negative slope as slowing down.</p>	<ul style="list-style-type: none"> <li>• <i>If a person walked for a kilometer at a steady speed, away from home, then turned around and walked back home at the same steady speed, what would the graph look like?</i></li> <li>• <i>How does the distance change during the second section of Tom's journey? What does this mean?</i></li> <li>• <i>How does the distance change during the last section of Tom's journey? What does this mean?</i></li> </ul>
<p><b>Student fails to mention distance or time</b></p> <p>For example: The student has not mentioned how far away from home Tom has travelled at the end of each section.</p> <p>Or: The student has not mentioned the time for each section of the journey.</p>	<ul style="list-style-type: none"> <li>• <i>Can you provide more information about how far Tom has travelled during different sections of his journey?</i></li> <li>• <i>Can you provide more information about how much time Tom takes during different sections of his journey?</i></li> </ul>
<p><b>Student fails to calculate and represent speed</b></p> <p>For example, the fails to answer (Q1i) or thinks that <math>7^{-1}</math> is the smallest number (possibly -7) in (Q2).</p>	<ul style="list-style-type: none"> <li>• <i>Can you provide information about Tom's speed for all sections of his journey?</i></li> <li>• <i>Can you write his speed as metres per second?</i></li> </ul>
<p><b>Student misinterprets the scale.</b></p> <p>For example: When working out distance the student has incorrectly interpreted the vertical scale as going up in tens rather than twenties</p>	<ul style="list-style-type: none"> <li>• <i>What is the scale on the vertical axis?</i></li> </ul>
<p><b>Student adds little explanation as to why the graph is or is not realistic</b></p>	<ul style="list-style-type: none"> <li>• <i>What is the total distance Tom covers? Is this realistic for the time taken? Why? Why not?</i></li> <li>• <i>Is Tom's fastest speed realistic? Is Tom's slowest speed realistic? Why? Why not?</i></li> </ul>





### 3.3 Answers

The answers to the card matching task can be found in the full lesson notes and are also provided here (below) in Figure 6.

Graph	Interpretation	Table	Graph	Interpretation	Table
A	5	P	B	10	S
C	4	V	D	2	Q
E	6	T	F	3	
G	1	W	H	8	R
I	7	U	J	9	X

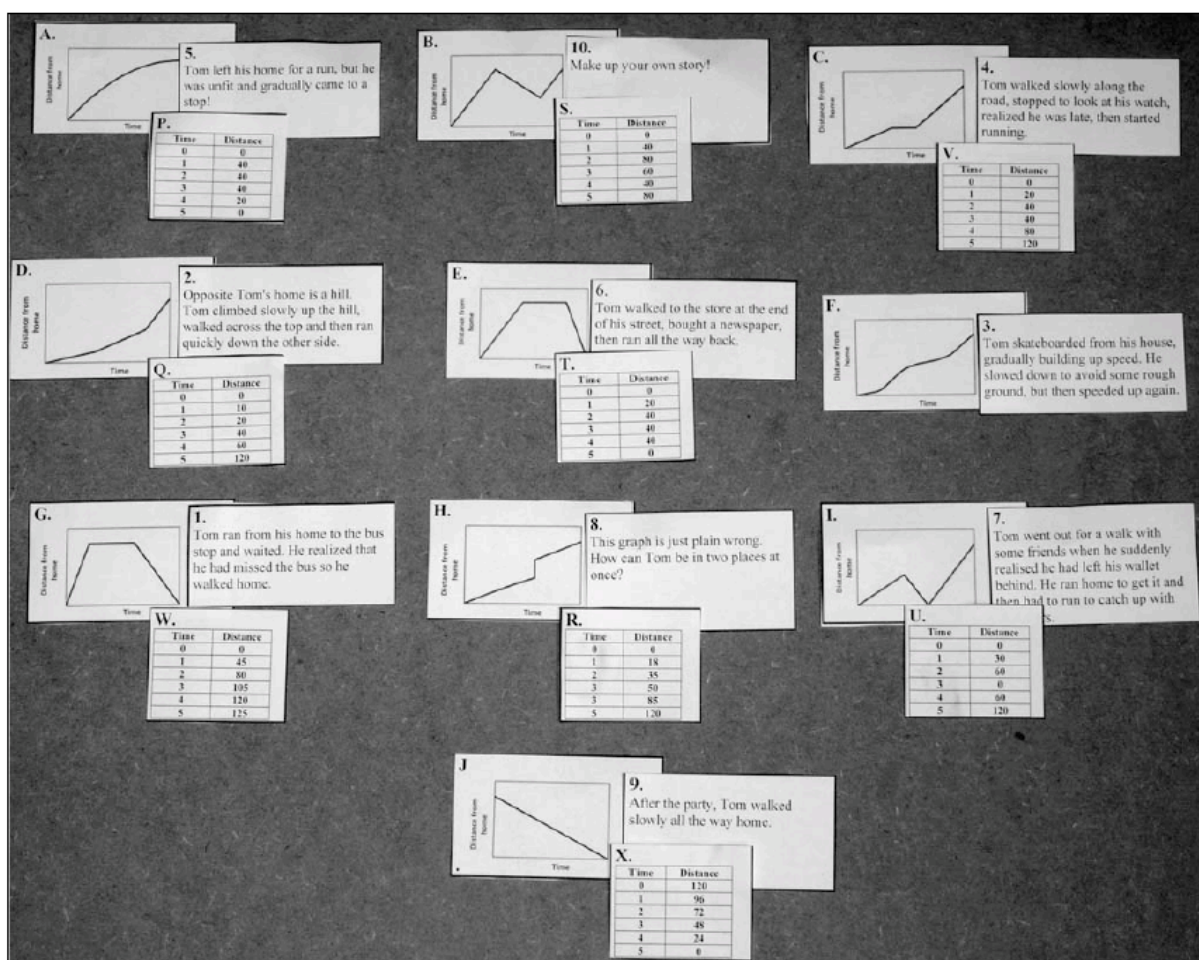


Figure 6: Answers to card matching activity