

Representing the relation between quantities



Can I sketch a graph based on a given situation?

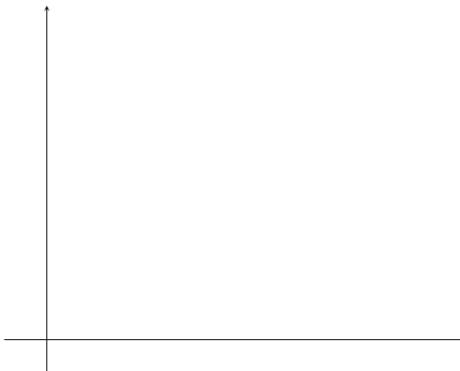
FaSMEd



Test:

Niklas gets on his bike and starts a ride from his home. Then he rides along the street with constant speed before it carves up a hill. On top of the hill, he pauses for a few minutes to enjoy the view. After that he drives back down and stops at the bottom of the hill.

Draw a graph to show how his speed changes as a function of the time.



1

Front 2

Front 1



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Can I sketch a graph based on a given situation?

Check your answer by circling either ✓ or ✗ for each statement:	How to continue?	
	Info	Practice
✓ I realized that the graph reaches the value of zero three times.	I1	P1
✗ I realized when the graph is increasing, decreasing, or remains constant.	I2	P2
✓ I realized that the graph is not always increasing and decreasing with the same speed.	I3	P3
✗ For example: The speed increases faster when Niklas is riding downhill than when he starts at home.		
✓ I realized that the graph has a different shape than the street with the hill at the end.	I4	P4
✗ I realized that there is only one value of speed related to any time in my graph and not more.	I5	P5
✓ I realized that the time is the independent variable, so it is recorded on the x-axis, and that the speed is the dependent variable recorded on the y-axis.	I6	P6

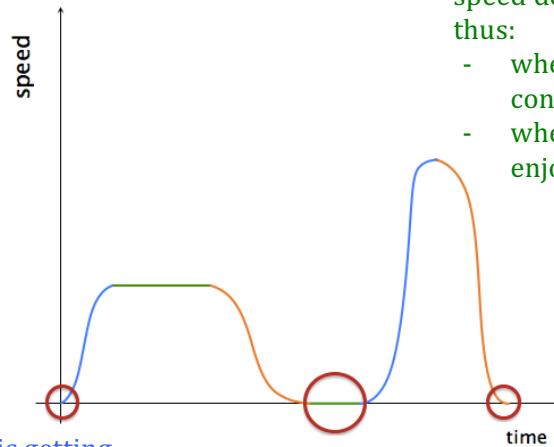
Your solution is correct if you always circle ✓! Continue with P7, P8 and E.

2

Your graph could for example look like this:

The graph reaches the value of zero when Niklas comes to a stop and therefore has a speed of 0 km/h, thus:

- at the very beginning.
- when he pauses on top of the hill.



The graph increases when Niklas is getting faster on his bike and therefore his speed is taking on larger values, thus:

- from the start until he reaches the constant speed with which he rides along the street.
- when he rides downhill.

The graph remains constant when Niklas' speed doesn't change for a period of time, thus:

- when he drives along the street with a constant speed.
- when he stands on top of the hill to enjoy the view.

The graph decreases when Niklas is slowing down and therefore his speed takes on smaller values, thus:

- when he rides uphill.
- after driving down the hill before he stops.

↑ Back 1

↓ Back 2



Can I sketch a graph based on a given situation?



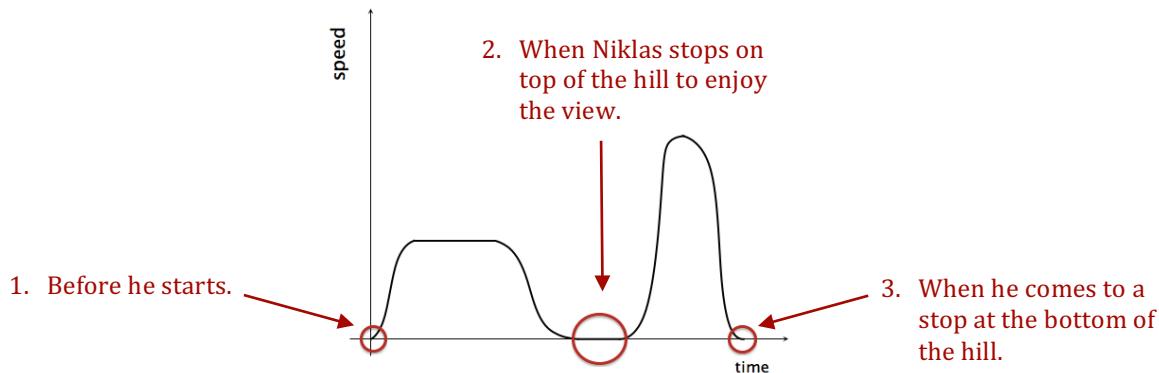
Info 1:

A graph reaches the **value of zero** every time when the dependent quantity, which means the quantity on the y-axis, reaches the value of zero. The graph is then touching the x-axis.

For example:

Whenever Niklas is not riding his bike, but stands still, the speed reaches a value of 0 km/h.

So for Niklas' ride the graph reaches the value of zero three times:



3

Front 4

Front 3



Can I sketch a graph based on a given situation?



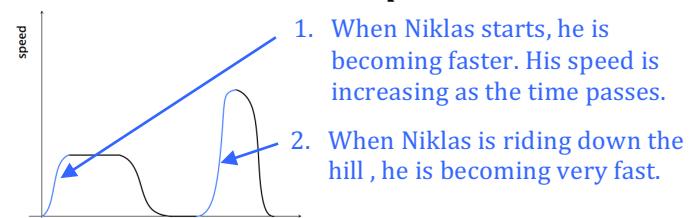
Info 2:

A graph **increases** when the values of the dependent quantity (y-axis) are becoming greater as the values of the independent quantity (x-axis) increase.

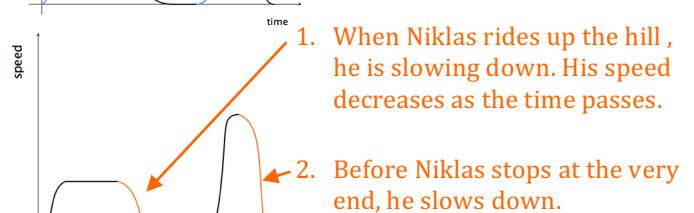
A graph **decreases** when the values of the dependent quantity (y-axis) are becoming smaller as the values of the independent quantity (x-axis) increase.

A graph **remains constant** when the values of the dependent quantity (y-axis) stay the same as the values of the independent quantity (x-axis) increase.

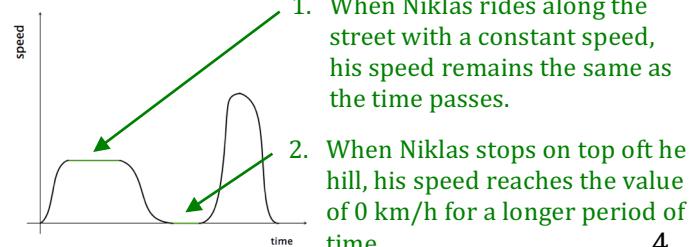
For example:



1. When Niklas starts, he is becoming faster. His speed is increasing as the time passes.
2. When Niklas is riding down the hill, he is becoming very fast.



1. When Niklas rides up the hill, he is slowing down. His speed decreases as the time passes.
2. Before Niklas stops at the very end, he slows down.



1. When Niklas rides along the street with a constant speed, his speed remains the same as the time passes.
2. When Niklas stops on top of the hill, his speed reaches the value of 0 km/h for a longer period of time.

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↑ Back 3

↓ Back 4

I3

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Can I sketch a graph based on a given situation?



Info 3:

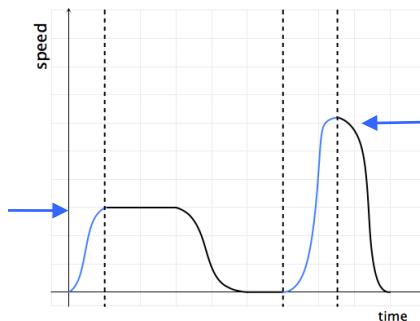
A graph can **increase or decrease more or less rapidly**.

This is due to the values of the dependent quantity (y-axis) changing more or less rapidly as the values of the independent quantity (x-axis) increase.

For example:

For Niklas' bike ride, the graph increases more rapidly when Niklas drives downhill compared to when he starts his ride at home. Because:

When Niklas starts his bike ride at home, he gets faster until he reaches the speed, with which he then drives along the street.



When Niklas drives downhill, he gets faster than in the beginning.
The speed increases very quickly.

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Front 6

Front 5

I4

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Can I sketch a graph based on a given situation?



Info 4:

A graph is **not like a picture of the given situation**, but represents the described relation between two quantities.

This is why you can see in a graph how the dependent quantity (y-axis) is changing with the independent quantity (x-axis).

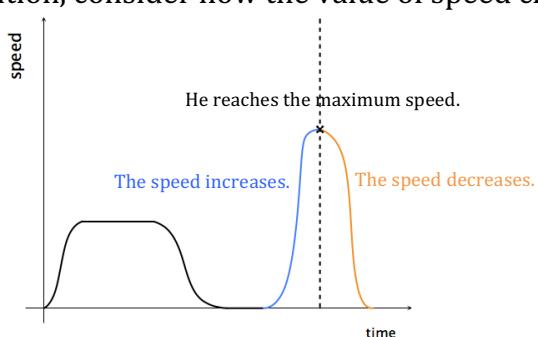
You can become aware of this as you look at single points of the graph and think about what they mean in the given situation. All those points together make up the graph.

For example:

Picture Niklas' bike ride at different times.

Consider the value of his speed at each of those moments.

In addition, consider how the value of speed changes before and after that moment.



When Niklas is riding downhill, his speed increases. After he reaches the maximum speed, the values of his speed decrease again.

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↑ Back 5

↓ Back 6



Can I sketch a graph based on a given situation?



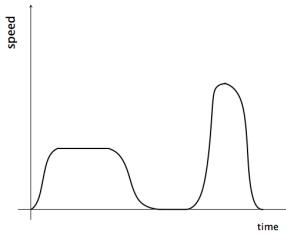
Info 5:

If there is **exactly one** value of the dependent quantity (y-axis) assigned to each value of the independent quantity (x-axis), you call the relation between the two quantities **functional**.

A graph that represents a functional relation between two quantities is called **unique**.

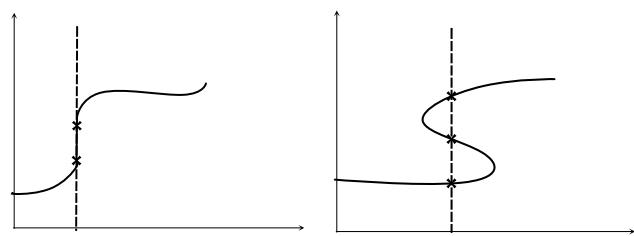
Example:

The graph of Niklas' bike ride is unique.
At any particular moment during the ride, you can measure exactly one value for his speed.
It is not possible for Niklas to have different speeds at the same time.
Thus, the relation between time and speed is functional.



Counterexample:

These two graphs are not representing functional relations between two quantities, because there is more than one value of the dependent quantity assigned to the same value of the independent quantity. The graphs are not unique. Thus, they can not represent the relation between time and speed.



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Front 8

Front 7



Can I sketch a graph based on a given situation?



Info 6:

If you want to represent the relation between two quantities in a graph, you have to enter the values of the quantities as points in a coordinate system.

To do this, you have to consider which quantity is the independent and which is the dependent one.

You always record the **independent quantity on the x-axis** and the **dependent quantity on the y-axis**.

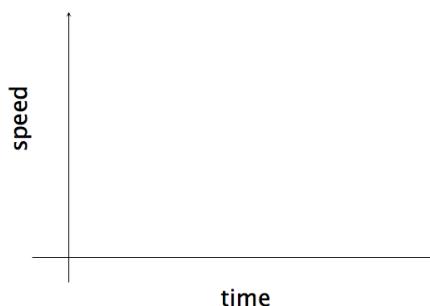
For example:

For Niklas' bike ride, the relation between two quantities is described: time and speed.

In the graph you want to show how the speed changes over time.

Therefore, the time is the independent quantity and the speed is the dependent quantity.

Thus, the time is recorded on the x-axis and the speed is recorded on the y-axis:



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↑ Back 7

↓ Back 8



Representing the relation between quantities

P1

Can I sketch a graph based on a given situation?



Practice 1:

Imagine that you want to draw a graph that shows how the speed changes as a function of the time based on the following situation.

At which times does the graph reach a value of zero? Tick the right boxes and explain your choices.

- Marie stands in front of the entrance after school and waits for her friend Jana.
- Slowly they start to walk home, because Marie is telling a joke.
- Both girls stop for a short while, since they have to laugh so much.
- After Jana says goodbye, Marie goes on more quickly.
- Because of a red traffic light, Marie has to stop at the next street.
- After it turns green, she starts to run, because she wants to catch up with her brother Ben.
- When Marie reaches Ben, she has to catch her breath for a short while.
- Finally they walk on together.

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Front 10

Front 9



Representing the relation between quantities

P2

Can I sketch a graph based on a given situation?



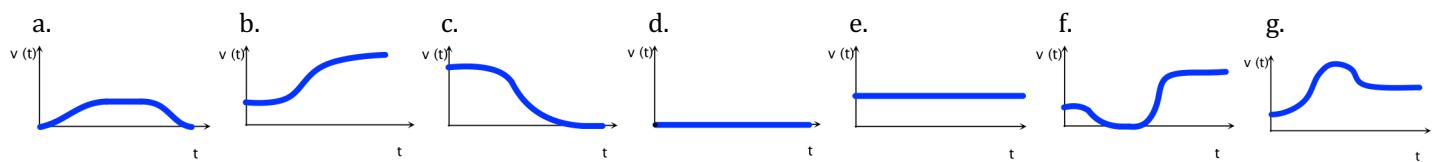
Practice 2:

The following situations describe different movements.

The graphs represent the speed $v(t)$ as a function of the time t .

Assign the right graph to each situation. Explain your choice each time.

1. You stand at the same spot during the entire time.
2. You ride down a hill and then alongside a river on your bike.
3. You drive in the car with your parents on a freeway. Your dad has to hit the break hard as you reach a traffic jam.
4. You run at approximately the same speed for the whole time.
5. A chain carousel starts slowly, circles around its own axis twice and then comes to a stop again.
6. Anna is walking to school. On the way she remembers that she forgot her maths folder at home. This is why she runs back home quickly.
7. Ilyas rides his bike to soccer practice. At a street corner, he stops to look at his watch. As he notices that he is running late, he has to hurry up now.



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The graph reaches a value of zero whenever the dependent quantity, in this case the speed, reaches the value of zero. That means every time Marie stands still and doesn't move. Then her speed is 0 km/h. This is why the graph reaches the value of zero when:

- Marie stands in front of the entrance after school and waits for her friend Jana.
- Slowly they start to walk home, because Marie is telling a joke.
- Both girls stop for a short while, since they have to laugh so much.
- After Jana says goodbye, Marie continues on faster than before.
- Because of a red light, Marie has to stop at the next street.
- She starts to run, after the light turns green, to reach her brother Ben.
- When Marie reaches Ben, she has to catch her breath for a short while.
- Finally they walk on together.

[Go back to the check and continue with the next point.](#)

↑ Back 9

↓ Back 10

These situations and graphs belong together:	Explanation:
1 – d	If you stand still, you have a speed of 0 km/h.
2 – g	When you ride downhill, your speed increases. Then you will get slower again and ride with approximately the same speed alongside the river.
3 – c	On the freeway you drive with a high speed. When you hit the break, the speed decreases quickly. If you are standing in a traffic jam, your speed is 0 km/h.
4 – e	When you walk with the same speed the whole time, the graph doesn't change but stays constant.
5 – a	At the very beginning, the carousel doesn't go with any speed as it stands. Then the speed increases. While it is turning, the speed stays approximately the same. Then the speed decreases again, when the carousel slows down. Finally the speed is 0 km/h, when the carousel comes to a stop.
6 – b	Anna is walking with a slow speed in the beginning. Then she starts running, so her speed increases.
7 – f	Ilyas is riding with a slow speed at the beginning. Then he stops to look at his watch, so his speed decreases to 0 km/h. Finally the speed increases quickly, because he has to hurry.

[Go back to the check and continue with the next point.](#)



Representing the relation between quantities

P3

Can I sketch a graph based on a given situation?



Practice 3:

Carlo pours himself a glass of water from a bottle.

At first he carefully pours a little water into the glass. Then he tilts over a lot of water at once until the glass is full.

While Carlo closes the bottle again with a lid, he leaves the full glass on a table.

Then he drinks half of the glass in small draughts taking a lot of time. He drinks the rest of the water almost at once.



- Draw a graph that shows how the filling height of the water in the glass changes as a function of the time.
- Describe at which periods of time the graph increases more or less rapidly and why.
- Describe at which periods of time the graph decreases more or less rapidly and why.

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Front 12

Front 11



Representing the relation between quantities

P4

Can I sketch a graph based on a given situation?

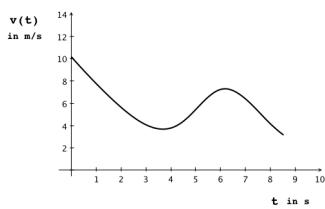


Practice 4:

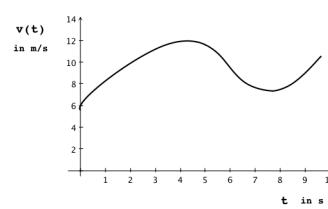
A skier goes down a slope (picture on the right).

- These graphs show his speed $v(t)$ (in meters per seconds) at each moment in time t (in seconds). Which graph belongs to the skier?

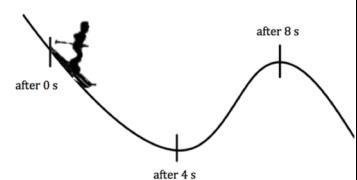
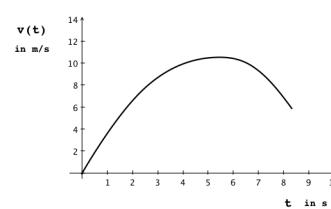
1.



2.

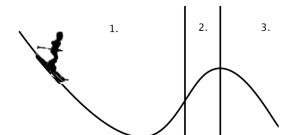


3.



- Look at the separate sections of the ski-run closely and describe for each section how the speed changes as the time progresses.

*In the 1. section, the skier runs _____ (uphill/ downhill) and is therefore going _____ (faster/ slower). The speed _____ (increases/ decreases).
In the 2. section ...*

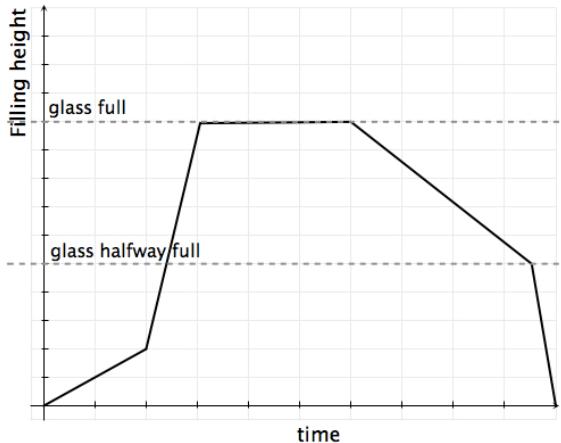


- What is the speed of the skier after 0 seconds, 4 seconds and 8 seconds?

- Look at your answers of the tasks a), b) and c) again. Did you choose the right graph in a)?

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a) Your graph could for example look like this:



b) When Carlo pours carefully at the beginning, the graph increases slowly. When he tilts over a lot of water at once after that, the glass fills up very quickly. That means the graph increases very quickly, because the filling height increases quickly.

c) When Carlo drinks half of the glass of water in small draughts, the filling height decreases slowly. This means the graph decreases slowly at first. At the end he drinks the other half of the water almost at once. Thus, the graph decreases very fast at the end.

[Go back to the check and continue with the next point.](#)

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 Back 12

a) The 2nd graph describes the ski-run.

b) In the first section, the skier runs downhill and is consequently going faster. His speed is increasing further.

In the second section, the skier runs uphill. Hence, he is slowing down. His speed decreases and so the value of $v(t)$ is getting smaller.

In the third section, the skier is running downhill once more. This is why he is getting faster. His speed increases and the value of $v(t)$ gets bigger again.

c) After 0 seconds: The speed $v(t)$ reaches a value of 6 m/s.

After 4 seconds: He is skiing with a speed of 12 m/s.

After 8 seconds: The speed of the skier is approximately 8 m/s.

d) The 2nd graph describes the ski-run correctly, because:

- it starts at a relatively high speed, because the skier runs downhill at the beginning,
- it increases to illustrate that the skier gets faster while running down the slope,
- it decreases to illustrate that the skier slows down while running uphill,
- it increases again to illustrate that the skier gets faster again since the slope goes downhill at the end.

[Go back to the check and continue with the next point.](#)



Practice 5:

For which of the following relations is it possible to draw a unique graph?

This means, there is exactly one value on the second axis assigned to each value on the first axis.

Explain your choice each time.

1. The distance from home on your way to school as a function of the time.
2. The body height depends on the shoe size.
3. The price for paint as a function of the amount of buckets (of paint) bought.
4. The breaking distance of a car depends on the driven speed.
5. The weight of a newborn child depends on its body height.
6. The monthly average temperature in a city as a function of the particular month.
7. The price of a book as a function of its number of pages.
8. The area of a square as a function of its edge length.
9. The number of students in a school depends on the number of teachers.
10. The time on a given day as a function of the currently measured temperature.

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Front 14

Front 13



Practice 6:

Imagine that you want to draw a graph for each of these relations.

Each time, decide which one of the following quantities you have to assign to the x-axis and which quantity relates to the y-axis:

temperature, distance, speed, time, pressure, concentration, money, weight.



1. In a prepaid contract for cell phones, the time left to make calls depends on the balance (prepaid).
2. The minimum distance from the car in front of you depends on your own speed.
3. The average temperature is determined every day for a month.
4. The more salt you put into the boiling water, the higher is the concentration of salt in your cooked pasta.
5. Tim's running speed determines the distance he can travel within half an hour.
6. The height of a skydiver is recorded every 2 seconds after he jumps out of the plane.
7. The weight of a parcel determines how much you have to pay for postage.
8. The distance of a boat to the coast depends on the time of measurement.
9. The deeper a diver submerges into the water, the greater is the water pressure.
10. The concentration of an ingested medication in the blood changes with the time after taking it.

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For these situations you can draw an ***unique*** graph:

No.	Explanation:
1	You can be in exactly one place at any given time.
3	You pay one assigned price for any chosen amount of buckets of wall paint.
4	The length of the breaking distance can be calculated exactly for any driven speed of a car.
6	You can assign exactly one average temperature for any month.
8	You can calculate exactly one area for every square with a particular edge length.

For these situations you can ***not*** draw an ***unique*** graph:

No.	Explanation:
2	Two people can have the same shoe size and still have different heights.
5	Two babies can have the same body height but still have various weights.
7	Two books with the same number of pages can still have different prices.
9	In two schools with a different number of students, can still work the same number of teachers.
10	One particular temperature can often be measured at different times during one day.

[Go back to the check and continue with the next point.](#)

 Back 13

 Back 14

No.	x-axis	y-axis
1.	money	time
2.	speed	distance
3.	time	temperature
4.	weight	concentration
5.	speed	distance
6.	time	distance
7.	weight	money
8.	time	distance
9.	distance	pressure
10.	time	concentration

[Go back to the check and continue with the next point.](#)



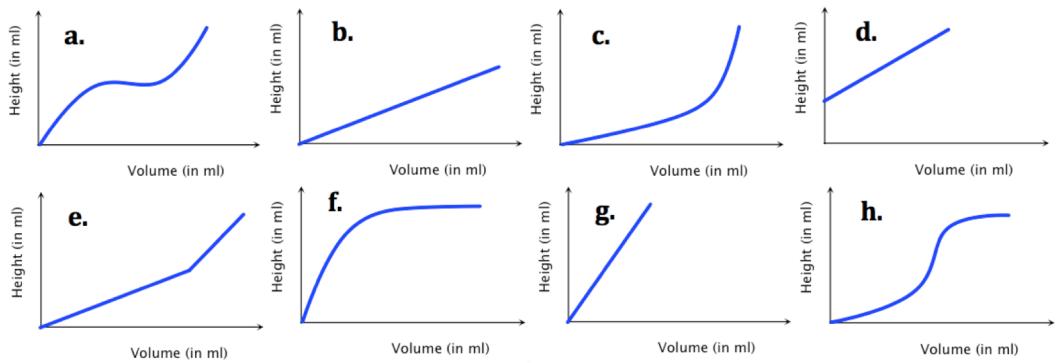
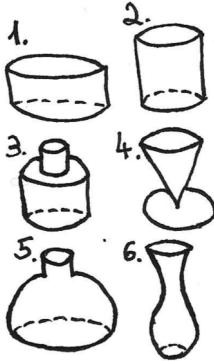
Can I sketch a graph based on a given situation?



Practice 7:

Amir performed an experiment at school: different beakers were filled up with water. He measured the filling height of the water (in cm) for different added volumes (in ml).

a) At home Amir wants to evaluate the experiment by drawing a „filling graph“ for each of the beakers. Unfortunately, he mixed up his results. Can you help him find the right graph for each beaker? Explain your choice each time.



b) Amir wants to repeat his experiment for this beaker.
Draw a suitable filling graph.



15

↓ Front 16

↑ Front 15

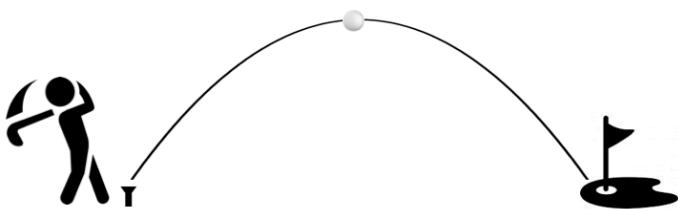


Can I sketch a graph based on a given situation?



Practice 8:

If a golf player hits the hole with only one stroke, you call his stroke an Ace.



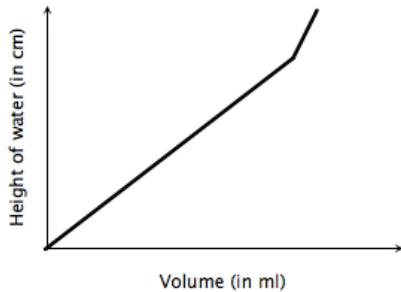
Draw a graph, for such a stroke, that shows what the distance of the golf ball from the Tee (point from which the ball is hit) is at any time after the player hits the ball.

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a)

These beakers and graphs belong together:	Explanation:
1 - b	The filling height increases slowly with the volume, because the beaker is very wide.
2 - g	The filling height increases quickly with the volume of water, because the beaker is thin.
3 - e	The bottom cylinder of the beaker is wider. Thus, the height of the water increases slowly at first and very fast after the water reaches the thin cylinder.
4 - f	The beaker is getting wider from bottom to top. So the filling height increases quickly at first and then more and more slowly as more water is added.
5 - c	The beaker is very wide at the bottom and gets thinner towards the top. Thus, the filling height increases very slowly at first and then faster and faster as more water is added.
6 - h	The beaker is wide at the bottom, is then getting thinner towards the top and finally wider again. So the filling height increases slowly at first, then faster and faster and finally slower again with the volume of the added water.

b)



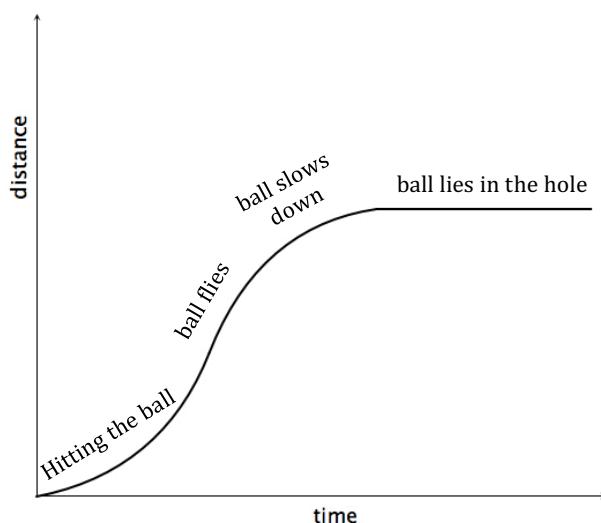
After finishing P7 and P8, you can continue with E.

↑ Back 15

↓ Back 16

The distance of the golf ball from the Tee is at first slowly, then faster and faster and at the end slowly again increasing.

As the golf ball hits the hole, its distance from the Tee remains constantly the same. Therefore, the graph must look like this:



After finishing P7 and P8, you can continue with E.



Can I sketch a graph based on a given situation?



Expand:

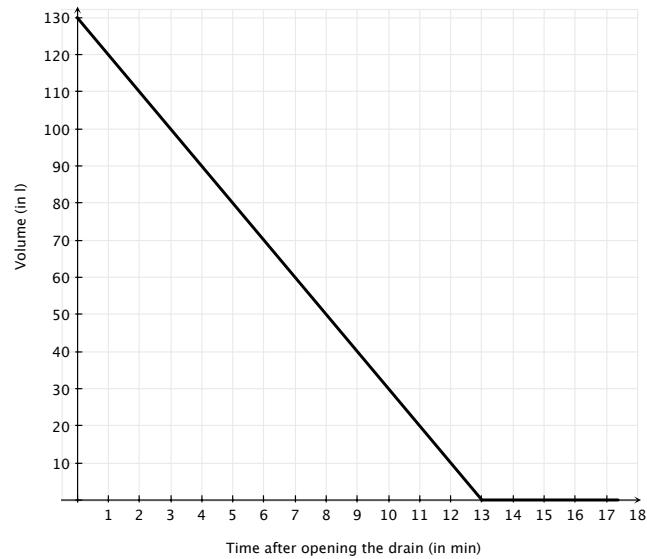
There are 130 liters (l) of water in a bath tub.

After opening the drain, 10 l of water run out of the tub every minute (min).



- a) Draw a graph that shows how many liters of water are in the tub at a certain time after opening the drain.
- b) Draw a graph that shows how the draining velocity of the water changes as a function of the time after opening the drain.

a)



b)

