FaSMEd

Raising Achievement through Formative Assessment in Science and Mathematics Education

Time-distance graphs – Part 1

Subject:	Mathematics
Age of students:	10-14 years
Hardware:	Tablets, pc, IWB or data-projector
Software:	IDM-TClass
Functionalities:	Sending and displaying Processing and analysing
Time:	4-6 hours
FaSMEd partner:	University of Turin
Short Abstract:	This activity is focused on **time-distance graphs**. It is aimed at gradually guiding students to the interpretation of a given time-distance graph, in relation to a story to which it refers.

Premises: theoretical tools

In presenting our methodology and the way of developing this activity we refer to two main theoretical tools.

The first theoretical tools are the Formative Assessment (FA) strategies proposed by Wiliam and Thompson (2007):

1) Clarifying/ Understanding/ Sharing learning intentions and criteria for success,
2) Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding,
3) Providing feedback that moves learners forward,
4) Activating students as instructional resources for one another,
5) Activating students as owners of their own learning.

The second theoretical tools are the Functionalities of Technology (FT) introduced within the FaSMEd Project (see the complete description on FaSMEd website https://microsites.ncl.ac.uk/fasmedtoolkit/theory-for-fa/the-fasmed-framework/):

(a) sending & displaying,
(b) processing & analysing,
(c) providing an interactive environment.

1. Content

This is the first part of an articulated activity on time-distance graphs. Students are asked to interpret a graph referring to a specific story. Verbal and graphical registers (Duval, 2006) are, therefore, involved. Since it is the first graph of this kind, students' interpretation of the graph is guided through gradual questions, posed to them within specific worksheets.

2. Activity

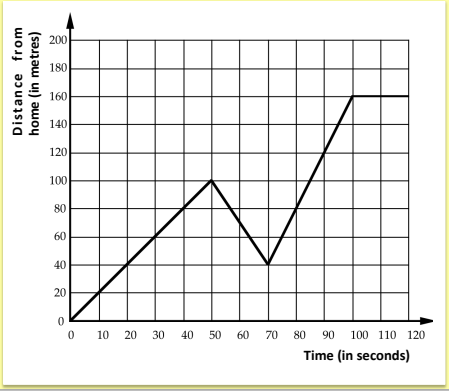
This activity is an adaptation from activities developed within the Mathematics Assessment Program (http://map.mathshell.org/materials/lessons.php). It can be developed referring to a set of *nine worksheets*.

2.1 The worksheets: focus and aims

As stated above, this set of worksheets is mainly aimed at guiding students to the interpretation of a graph, according to a story.

Worksheet 1 introduces the graph and the corresponding story: the graph represents the way in which a student, Tommaso, walks, on a straight road, from home to the bus stop. The question posed to students makes them focus on the second section of the graph, that is the segment that connects the points (50,100) and (70,40). Students are asked to deduce, from the graph, what happens during the period of time from 50s to 70s.

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



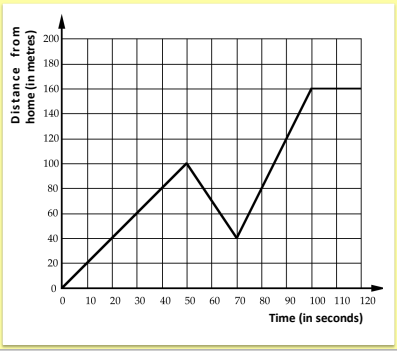
(1) What happens in the period of time between 50s and 70s? How do you know it?

Fig. 1: Worksheet 1

The question in *Worksheet 1* is therefore aimed at making students focus on the interpretation an ascending/descending line within a time-distance graph. Moreover, students are asked to explain how they deduced this information from the graph, in order to make them reflect on the reasons supporting the correct interpretation of a time-distance graph.

Worksheet 1A is a helping worksheet. It could be sent to the students that face difficulties in facing the question within worksheet 1.

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



(1) What happens in the period of time between 50s and 70s? How do you know it?

Help to answer to question 1:

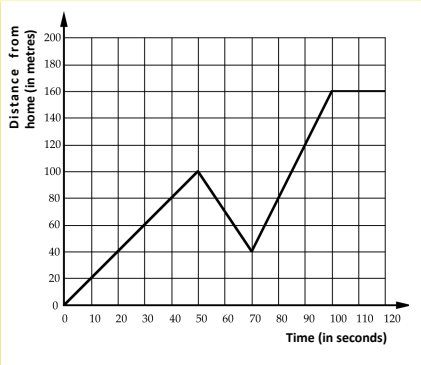
Remember that Tommaso is walking on a straight road.
What is his distance from home after 50s?
What is his distance from home after 70s?"

Fig. 2: Worksheet 1A

The "help" within *worksheet 1A* aims at supporting the students in the interpretation of the graph in two ways. On one side, the suggestion within the worksheet ("Remember that Tommaso is walking on a straight road") aims at preventing students from confusing the graph with the drawing of the road (proposing interpretations such as "Tommaso turns right, then left" or "Tommaso is downhill and then up again"). On the other side, the two questions within worksheet 1A makes the students focus on the way in which Tommaso's distance from home varies, helping the students observe that, since the distance is decreasing, Tommaso is approaching home. Moreover, students are led to the interpretation of a point of the graph as "bearer" of two pieces of information (the distance from home and the time spent).

Worksheet 1B is a worksheet prompting a poll: three answers, given by other fictitious students, are proposed, with the request of identifying the correct one.

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



(1) What happens in the period of time between 50s and 70s? How do you know it?

What is the correct answer?

(a) In the period from 50s to 70s, Tommaso comes back.

(b) In the period from 50s to 70s, Tommaso changes his road.

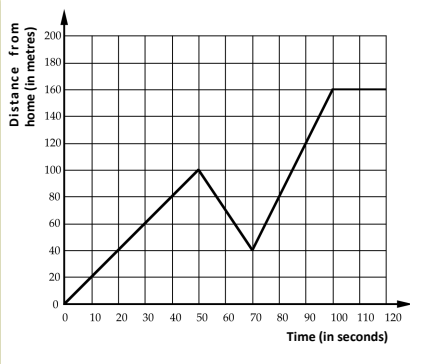
(c) In the period from 50s to 70s, the road, on which Tommaso is walking, goes down.

Fig. 3: Worksheet 1B

Worksheet 1B could be proposed as a starting point of the discussion on worksheet 1 to focus on the possible typical mistakes in the interpretation of a time-distance graph as a drawing of the road or of a hill.

Worksheet 2 presents the second question through which students are guided in the interpretation of the given graph. Here the focus is on the last section of the graph, that is the horizontal segment (100,160)-(120,160).

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



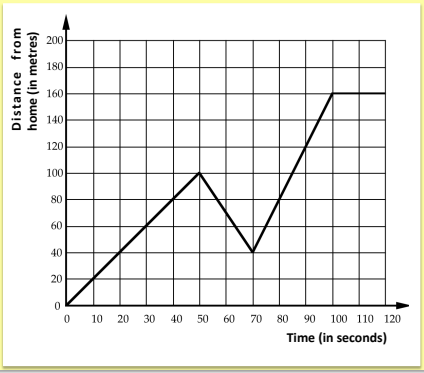
(2) What happens during the last 20s? How did you establish it?

Fig. 4: Worksheet 2

The question in *Worksheet 2* is focused on the interpretation of a horizontal line within a time-distance graph. Moreover, as for worksheet 1, students are asked to explain how they deduced this information from the graph, in order to make them reflect on the reasons supporting the correct interpretation of a time-distance graph.

Worksheet 2A is a worksheet prompting a poll: three justifications, given by fictitious students, are proposed, with the request of identifying the most complete one among these.

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



(2) What happens during the last 20s? How did you establish it?

(a) During the last 20s, Tommaso is not walking because we have already said that he has reached the bus stop.

(b) I think that, during the last 20s, Tommaso is not walking because, from the graph, it is possible to understand that, in the period between 100s and 120s, he is always at the same distance from home, that is 160m.

(c) I understood that, during the last 20s, Tommaso is not walking because the line of the graph is horizontal.

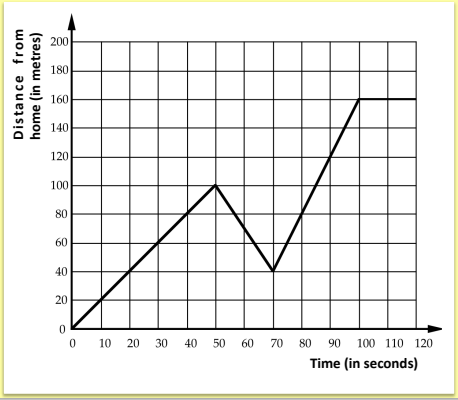
Some students of another class wrote these answers. Which of them is the most complete?

Fig. 5: Worksheet 2A

Through the discussion on the results of the poll within *worksheet 2A*, it is possible to make students reflect on the meaning of the term "complete" when this term is used to evaluate the justification to a given answer. The discussion can enable to highlight the characteristics of "complete justification" in terms of correctness, clearness and with reference to the mathematical reasons underlying the choice of a specific answer.

Worksheet 3 requires students to determine when Tommaso reaches the bus stop. Here the focus is therefore on the interpretation of a point in a time-distance graph as a bearer of two pieces of information: the distance from home and the time spent. Students have to identify the point (100,160) as the one on which they have to focus in order to find the answer.

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



(3) After how many seconds does Tommaso reach the bus stop?

(a) After 120s;

(b) After 50+70+100+120 seconds, that is after 340 seconds;

(c) After 100 seconds;

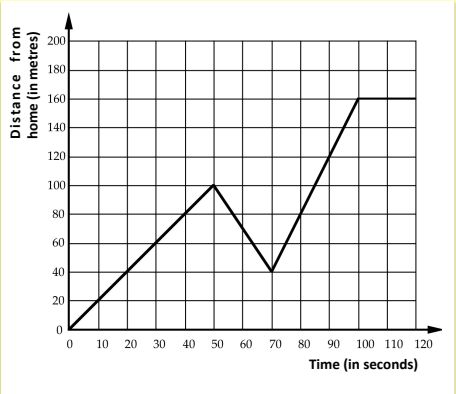
(d) After 50 seconds.

Fig. 6: Worksheet 3

The question in *worksheet 3* is proposed as a poll. The first option represents one of the typical mistakes made by students, who interpret the last point on the right of the graph as the one representing the moment in which Tommaso stops. The second option was inserted to see if students would have chosen it because of the "mathematical expression" proposed, without analysing its correctness. This poll is conceived as a starting point for a discussion focused on the reason underlying the choice of the answers.

Worksheet 4 is the last question proposed to support students' interpretation of the graph. It makes students focus on the distance walked by Tommaso to reach the bus stop.

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



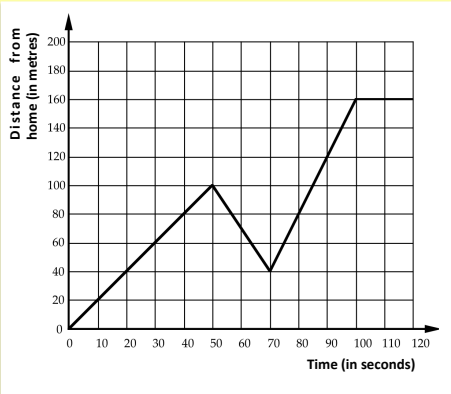
(4) Does Tommaso walk for 160m? Why?

Fig. 7: Worksheet 4

The question aims at making the students reflect one the difference between two concepts: the distance from home and the distance that was walked through. Again, students are also asked to share the reasons underlying their answers.

Worksheet 4A is a helping worksheet to be sent to those students who have difficulties in facing worksheet 4.

Every morning Tommaso walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



(4) Does Tommaso walk for 160m? Why?

Help to answer to question 4:

Analyse the graph and answer to the following questions:

a) What is the distance that Tommaso walked during the initial 50s?	**Answer:**
b) What is the distance that Tommaso walked in the period of time from 50s to 70s?	**Answer:**
c) What is the distance that Tommaso walked in the period of time from 70s to 100s?	**Answer:**
d) What is the distance that Tommaso during the last 20s?	**Answer:**

Answer to question 4:

Fig. 8: Worksheet 4A

The "help" within *worksheet 4* consists of four different questions through which students are guided to focus, separately, on the different sections of the graph. In this way, they can determine the distance walked by Tommaso as the sum of the distances walked by Tommaso during the periods of time corresponding to each section of the graph.

Worksheet 5 focuses on a global interpretation of the graph. Students are asked to propose a possible completion of the story, in tune with the interpretation of the graph that the previous worksheets supported.

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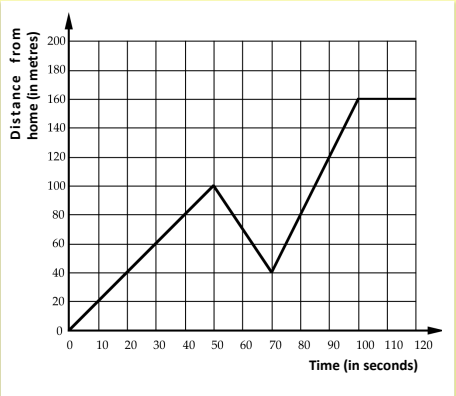

(5) After having answered to the questions in the previous worksheets, describe how Tommaso has walked on the road from his home to the bus stop. What could have happened to him?

Fig. 9: Worksheet 5

Worksheet 5 is aimed at enabling the students to recall all the aspects highlighted in the previous worksheets and corresponding discussions.

2.2 Methodology

Our hypothesis is that, in order to raise students' achievement, Formative Assessment (FA) has to focus not only on basic competences, but also on metacognitive factors (Schoenfeld, 1992). Accordingly, we planned and developed class activities with the aim of: (a) fostering students' development of ongoing reflections on the teaching-learning processes; (b) focusing on making thinking visible (Collins, Brown & Newmann, 1989), through the sharing of their ideas with the teacher and the classmates.

For this reason, we suggest that, during the activities, the teacher guides the students to focus on the analysis and comparison of not only their *products* but also the *processes* that led to these products. In particular, the class should be led to discuss, on one side, the written productions and, on the other side, the strategies developed to carry out the tasks.

As regards the collective analysis of the students' written productions and the developed strategies, in particular, we refer to *argumentation* as a possible FA tool in the interaction between teacher and students. Specifically, argumentation is promoted to support the development of effective class discussions, starting from questions such as: "Explain what you did", "Explain why your approach is effective", and to guide students in assessing the correctness, the clearness and the completeness of given explanations (their own or others).

The methodology adopted is in tune with these hypotheses. It will be clarified in section 2.4, after the introduction of the technology used (section 2.3).

2.3 Technology

In tune with the hypotheses presented in the previous section, we explored the use of a CCT, which connects the students' tablets with the teachers' laptop and allows the students to share their productions, and the teacher to easily collect the students' opinions and reflections during or at the end of an activity: IDM-TClass.

In the use of IDM-TClass to support FA processes, we in particular focused on the following three main functions of this software:

- the possibility of distributing documents to students and collecting documents from the students' tablets (related to the functionality *Sending and Displaying*);
- the possibility of creating instant polls and immediately showing their results to the whole class (related to the functionality *Processing and Analysing*);
- the possibility of displaying the students' written productions through the data projector or the interactive whiteboard (related to the functionality *Sending and Displaying*).

Each school was provided with tablets for the students and computers for the teachers, linked to IWB or data projector. In order to foster collaboration and sharing of ideas, students were asked to work in pairs or in small groups on the same tablet.

2.4 Structure of a typical lesson and Aspects of Formative Assessment

In the following, we present the typical structure of a lesson developed during the teaching experiments carried out in Italy, in this case with specific reference to worksheets 1, 1A, 1B, 2, 2A, 3, 4, 4A, 5.

Usually the activity starts with a worksheet focused on one or more questions (in this case **worksheets 1**, **2**, **4** and **5**), sent from the teacher's laptop to the students' tablets (functionality *Sending and Displaying*). Students work in pairs or small groups of three.

After facing the task and answering the questions, the pairs/groups send back their written productions (functionality *Sending and Displaying*) to the teacher. The teacher can decide to send helping worksheets (*FA strategy 3*, aimed at the activation of *FA strategy 5*) to some groups, or the groups can ask for them. In this case, the helping worksheet **1A** and **4A** could be sent to support the students in answering to the questions on the worksheets 1 and 4.

After all groups have sent back their answers, the teacher sets up a classroom discussion (*FA strategy 2*) in which the students' written productions are shown (functionality *Sending and Displaying*) and feedbacks are given by the teacher and by classmates (*FA strategies 3 and 4*, aimed at the activation of *FA strategy 5*). The discussion is engineered starting from the teacher's selection of some of the received written answers, shown on the IWB. The discussion aims at highlighting (*FA strategy 3*): (a) typical mistakes; (b) effective ways of processing the tasks; (c) the comparison between the different ways of justifying claims. In this, the criteria for success could be clarified through the analysis and comparison of the different written productions (*FA strategy 1*).

The worksheets involving the activation of polls (functionality *Processing and Analysing*) could be used to focus the discussion (*FA strategy 2*, that could lead to the activation of other FA strategies, such as 3, 4, 5) during different parts of the lessons. In this case the worksheets aimed at prompting polls are 1B, 2A and 3.

3. Further Information

We recommend that, when the teacher introduces the worksheets that are going to be sent to the students, she stresses some aspects. This is especially crucial with younger students (grade IV and V).

As regards **worksheets 1**, **2**, **3** and **4** (those aimed at guiding students to the gradual interpretation of the graph), it is important, when the worksheets are displayed before the working group phase, to help students identify the sections of the graph on which to focus when answering the different questions.

During the teaching experiments we carried out, we used **worksheets 1B** very few times because the discussion on worksheets 1 and 1A enabled also to focus on the typical mistakes related to the interpretation of a time-distance graph as the drawing of a road or of a hill.

Since the polls do not enable to add written justifications to the provided answers, the discussion on **worksheet 3** has to focus especially on the reasons underlying the choice of a specific option (in particular the correct one). An important aspect to be stressed is again the interpretation of a point of a time-distance graph as "bearer" of two pieces of information (the distance from home and the time spent).

The discussion on **worksheet 5** could help students consolidate their competences in the interpretation of a time-distance graph. In particular, students should be guided to reflect on the fact that a correct story should involve two instantaneous sudden changes in Tommaso's direction, corresponding to the point (50,100) and (70,40) within the graph.

4. References

Collins, A., Brown, J.S., & Newman, S.E. (1989). Cognitive Apprenticeship: Teaching the Crafts of Reading, Writing and Mathematics! In L.B. Resnick (Ed.), *Knowing, Learning, and Instruction: Essays in Honor of Robert Glaser* (pp. 453-494). Hillsdale, NJ: Lawrence Erlbaum Associates.

Cusi, A., Malara, N.A., & Navarra, G. (2011). Early Algebra: Theoretical Issues and Educational Strategies for Bringing the Teachers to Promote a Linguistic and Metacognitive approach to it. In J. Cai, & E.J. Knuth (Eds.), *Early Algebraization: Cognitive, Curricular, and Instructional Perspectives* (pp. 483-510). Berlin Heidelberg: Springer.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 103–131.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Wiliam, D., & Thompson, M. (2007). Integrating assessment with instruction: What will it take to make it work? In C. A. Dwyer (Ed.), *The future of assessment: Shaping teaching and learning* (pp. 53–82). Mahwah, NJ: Erlbaum.

Further information about the software IDM-TClass can be found on the webpage
http://www.tecnilabedu.com/prodotto05EN.html